Neutrino oscillation data revisited in the minimal SO(10) model

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based on the work in collaboration with
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Work in progress, arXiv:0906.xxxx, to appear.

Brief review of neutrino oscillation data

- Now neutrino oscillation is in the era of precision measurement. A global fit of all the neutrino data show the following values for the oscillation parameters. G. Fogli et al., PRL101:141801 (2009)

$$\Delta m_{23}^2 = 2.39^{+0.42}_{-0.33} \times 10^{-3}\mathrm{eV}^2$$

$$\Delta m_{12}^2 = 7.67^{+0.52}_{-0.53} \times 10^{-5}\mathrm{eV}^2$$

$$\sin^2\theta_{23} = 0.466^{+0.178}_{-0.135}$$

$$\sin^2\theta_{12} = 0.312^{+0.063}_{-0.049}$$

$$\sin^2\theta_{13} \le 0.061$$

- This data shows that **lepton sector have 2 large mixing angles plus 1 small angle.** This is very contrary to the quark sector as **quark sector have 3 small mixing angles.**
- This may forecast us some difficulty to build a unified model of flavor which describes both quarks and leptons.

“Hints of $\theta_{13} > 0$ from global neutrino data analysis”

G. Fogli et al., PRL101:141801 (2009)

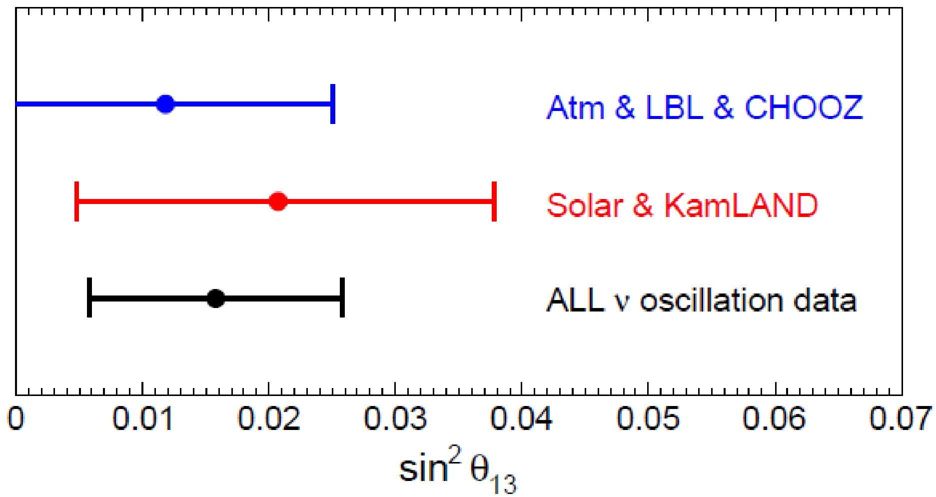
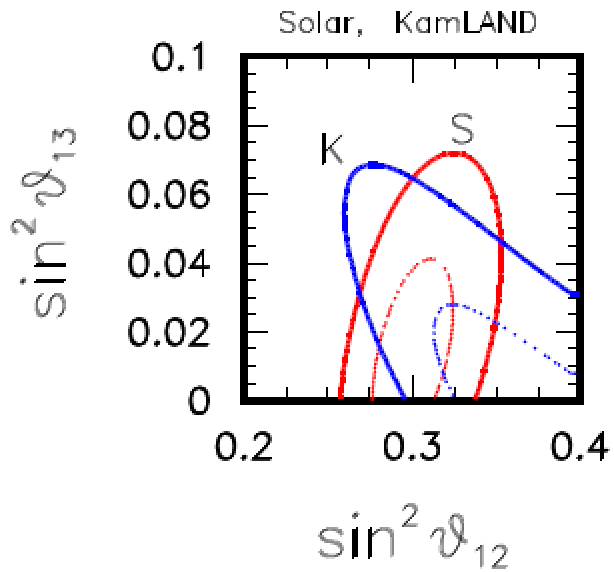


FIG. 2: Global ν oscillation analysis: Allowed 1σ ranges of $\sin^2\theta_{13}$ from different input data.



The best fit value is $\sin^2\theta_{13} = 0.016 \pm 0.010$ $(1\sigma, \text{all } \nu \text{ data})$

This may suggest some hints or exclude some GUT models.

Predictions on θ_{13} for various SO(10) GUT models

C. Albright and M. C. Chen, PRD74:113006 (2006)

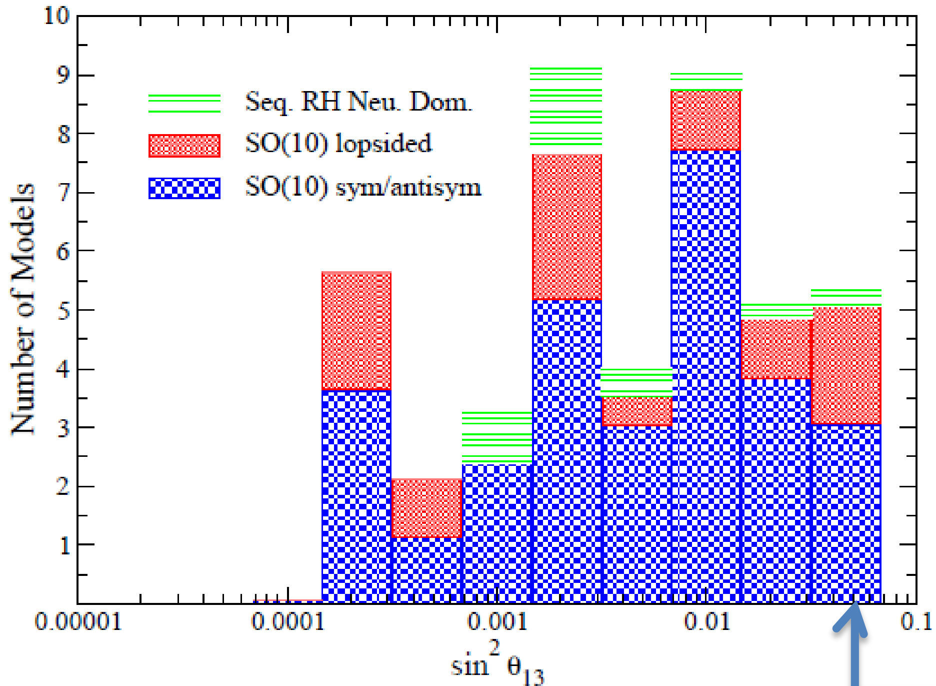


Figure 2: Predictions of $\sin^2\theta_{13}$ for the SO(10) models considered.

$\sin^2\theta_{13} = 0.016$

Many SO(10) GUT models predict non-zero and relatively large θ_{13}

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Minimal SU(5) GUT

Matter multiplets: $\{\mathbf{10}+\bar{\mathbf{5}}+\mathbf{1}\}$ still non trivial anomaly cancellation

$$\mathbf{10}:\begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}$$

$$\bar{\mathbf{5}}:(d_1^c, d_2^c, d_3^c, e, -\nu_e)$$

$$\mathbf{1}:\nu^c$$

Higgs: $\mathbf{24}_H, \quad \{\mathbf{5}_H, \quad \bar{\mathbf{5}}_H\}$

Yukawa Couplings: $Y_u^{ij}\mathbf{10}_i\mathbf{10}_j\mathbf{5}_H + Y_d^{ij}\mathbf{10}_i\bar{\mathbf{5}}_j\bar{\mathbf{5}}_H$

$$M_\ell = M_d^T \Rightarrow m_b = m_\tau, m_s = m_\mu, m_d = m_e$$

The 3rd generation may be good, but for the 1st and the 2nd generations it gives no realistic mass spectra!

Matter Unification in 16 of SO(10)

u_1	:	$\vert\uparrow\downarrow\uparrow\uparrow\downarrow>$
u_2	:	$\vert\uparrow\downarrow\uparrow\downarrow\uparrow>$
u_3	:	$\vert\uparrow\downarrow\downarrow\uparrow\uparrow>$
d_1	:	$\vert\downarrow\uparrow\uparrow\uparrow\downarrow>$
d_2	:	$\vert\downarrow\uparrow\uparrow\downarrow\uparrow>$
d_3	:	$\vert\downarrow\uparrow\downarrow\uparrow\uparrow>$
u_1^c	:	$\vert\downarrow\downarrow\uparrow\downarrow\downarrow>$
u_2^c	:	$\vert\downarrow\downarrow\downarrow\uparrow\downarrow>$
u_3^c	:	$\vert\downarrow\downarrow\downarrow\downarrow\uparrow>$
d_1^c	:	$\vert\uparrow\uparrow\uparrow\downarrow\downarrow>$
d_2^c	:	$\vert\uparrow\uparrow\downarrow\uparrow\downarrow>$
d_3^c	:	$\vert\uparrow\uparrow\downarrow\downarrow\uparrow>$
ν	:	$\vert\uparrow\downarrow\downarrow\downarrow\downarrow>$
e	:	$\vert\downarrow\uparrow\downarrow\downarrow\downarrow>$
e^c	:	$\vert\downarrow\downarrow\uparrow\uparrow\uparrow>$
ν^c	:	$\vert\uparrow\uparrow\uparrow\uparrow\uparrow>$

Minimal SO(10) Model

K. Babu and R. Mohapatra, PRL.70:2845-2848 (1993)

Quarks and leptons ~{*16*$_i$} Automatic anomaly cancellation

Contains ν_R and seesaw mechanism

Natural framework for explaining neutrino data

Minimal SO(10) Model: renormalizable Yukawa coupling model

Higgs: $\{\mathbf{210}_H + \overline{\mathbf{126}}_H + \mathbf{126}_H + \mathbf{10}_H\}$

Automatic R-parity

$$\mathcal{L}_{\text{Yukawa}} = f_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H + h_{ij}\mathbf{16}_i\mathbf{16}_j\overline{\mathbf{126}}_H$$

Under $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(4)_C$

$$\overline{\mathbf{126}} = (\mathbf{1},\mathbf{3},\mathbf{10}) + (\mathbf{3},\mathbf{1},\overline{\mathbf{10}}) + (\mathbf{1},\mathbf{1},\mathbf{6}) + (\mathbf{2},\mathbf{2},\mathbf{15})$$

$(1,1,0)$ of SM $\rightarrow$ gives mass of ν_R

related

$M_{\nu_R} \leftrightarrow M_{q,l}$

$(1,2,1/2)$ of SM $\rightarrow$ Higgs doublet

Fermion Mass matrices

- In minimal SO(10) model, all the fermions mass matrices are written in terms of two basic mass matrices

$$\begin{aligned} M_u &= c_{10} M_{10} + c_{126} M_{126} \\ M_d &= M_{10} + M_{126} \\ M_{\nu_D} &= c_{10} M_{10} - 3 c_{126} M_{126} \\ M_e &= M_{10} - 3 M_{126} \\ M_{\nu_R} &= c_R M_{126} \end{aligned}$$

Model has only 11 real parameters plus 7 phases

- A factor -3 is the CG coefficient, and it plays an important role to explain the mass difference between quarks and leptons.
- This leads to the GUT relation between quarks and leptons.

$$M_e = c_d \left(M_d + \kappa M_u \right) \text{ in SO(10)} \quad \Longleftrightarrow \quad M_\ell = M_d^T \text{ in SU(5)}$$

- By putting the experimental data of quark and charged leptons, we can fix the GUT parameters c_d and κ, and get a definite prediction in the neutrino sector!

Previous estimation of quark and lepton masses

- MSSM with $\tan\beta = 10$ Koide and Fusaoka, PRD57:3986-4001 (1998)

SUSY threshold corrections are not included.

$$m_u(M_Z) = 2.33^{+0.42}_{-0.45}\ \text{MeV} \to m_u(M_G) = 1.04^{+0.19}_{-0.20}\ \text{MeV}$$

$$m_d(M_Z) = 4.69^{+0.60}_{-0.66}\ \text{MeV} \to m_d(M_G) = 1.33^{+0.17}_{-0.19}\ \text{MeV}$$

$$m_s(M_Z) = 93.4^{+11.8}_{-13.0}\ \text{MeV} \to m_s(M_G) = 26.5^{+3.3}_{-3.7}\ \text{MeV}$$

$$m_c(M_Z) = 0.677^{+0.056}_{-0.061}\ \text{GeV} \to m_c(M_G) = 0.302^{+0.025}_{-0.027}\ \text{GeV}$$

$$m_b(M_Z) = 3.00 \pm 0.11\ \text{GeV} \to \boxed{m_b(M_G) = 1.00 \pm 0.04\ \text{GeV}}$$

$$m_t(M_Z) = 181 \pm 13\ \text{GeV} \to m_t(M_G) = 129^{+196}_{-40}\ \text{GeV}$$

$$m_e(M_Z) = 0.48684727 \pm 0.00000014\ \text{MeV}$$

$$\to m_e(M_G) = 0.32502032 \pm 0.00000009\ \text{MeV}$$

$$m_\mu(M_Z) = 102.75138 \pm 0.00033\ \text{MeV}$$

$$\to m_\mu(M_G) = 68.59813 \pm 0.00022\ \text{MeV}$$

$$m_\tau(M_Z) = 1746.7 \pm 0.3\ \text{MeV} \to \boxed{m_\tau(M_G) = 1171.4 \pm 0.2\ \text{MeV}}$$

Bottom-tau mass unification shows a good agreement

Updated estimation of quark and lepton masses

- MSSM with $\tan\beta = 10$ Z, Z. Xing, et al., PRD77:113016 (2008)

SUSY threshold corrections (non-holomorphic Yukawa corrections) are included.

$$m_u(M_Z) = 1.27^{+0.50}_{-0.42}\ \text{MeV} \to m_u(M_G) = 0.49^{+0.20}_{-0.17}\ \text{MeV}$$

$$m_d(M_Z) = 2.90^{+1.24}_{-1.19}\ \text{MeV} \to m_d(M_G) = 0.70^{+0.31}_{-0.30}\ \text{MeV}$$

$$m_s(M_Z) = 55^{+16}_{-15}\ \text{MeV} \to m_s(M_G) = 13 \pm 4\ \text{MeV}$$

m_s becomes small

$$m_c(M_Z) = 0.619 \pm 0.084\ \text{GeV} \to m_c(M_G) = 0.236^{+0.037}_{-0.036}\ \text{GeV}$$

$$m_b(M_Z) = 2.89 \pm 0.09\ \text{GeV} \to m_b(M_G) = 0.79 \pm 0.04\ \text{GeV}$$

$$m_t(M_Z) = 171.7 \pm 3.0\ \text{GeV} \to m_t(M_G) = 92.2^{+9.6}_{-7.8}\ \text{GeV}$$

m_b becomes small

$$m_e(M_Z) = 0.486570161 \pm 0.000000042\ \text{MeV}$$

$$\to m_e(M_G) = 0.283755495^{+0.000000024}_{-0.000000025}\ \text{MeV}$$

$$m_\mu(M_Z) = 102.7181359 \pm 0.0000092\ \text{MeV}$$

$$\to m_\mu(M_G) = 59.9033617 \pm 0.0000054\ \text{MeV}$$

Not so good agreement

$$m_\tau(M_Z) = 1746.24^{+0.20}_{-0.19}\ \text{MeV} \to m_\tau(M_G) = 1021.95^{+0.11}_{-0.12}\ \text{MeV}$$

Input data of quarks and charged leptons

- CKM mixing angles and a phase

Z, Z. Xing, et al., *private communications.*

$$V_{us} = 0.2257 \pm 0.0021 \to V_{us} = 0.2257 \pm 0.0021$$
$$V_{ub} = 0.00431 \pm 0.0003 \to V_{ub} = 0.0038 \pm 0.0002$$
$$V_{cb} = 0.0416 \pm 0.0006 \to V_{cb} = 0.0371 \pm 0.0005$$
$$\sin(2\beta) = 0.687 \pm 0.032 \to \sin(2\beta) = 0.688 \pm 0.033$$

- Number of parameters

$c_{10},\ c_{126},\ M_{10}\ M_{126} \to 2+2+3+12=19$
We restrict all the masses to be real, but allow the sign ambiguities, then the numbers of parameters are decreased by 5
$\to 19-5=14$

- 13 inputs for the charged fermion data

⇒ fix most parameters in the model; **only one parameter left free!**

⇒ **high predictivity on the neutrino sector!**

Predictions for the Neutrino mass matrices

- Mass relation for the neutrino sector:

$$M_D = M_u + \frac{1+3/c_d}{\kappa}(M_d - M_e)$$

$$M_R = c_R (M_d - M_e)$$

- Type-I See-saw mass matrix: $M_\nu = -M_D M_R^{-1} M_D$

$$M_\nu = -M_D M_R^{-1} M_D$$
$$= \frac{4}{c_R}\left[\left(\frac{\kappa}{1+3/c_d}\right)^2 (M_d - M_e)\right.$$
$$\left. -2\left(\frac{\kappa}{1+3/c_d}\right) M_u + M_u (M_d - M_e)^{-1} M_u\right]$$

- ***Neutrino mass matrix is determined only by the well-known charged fermion mass matrices!***

Neutrino mass matrix in detail

- Type-I See-saw mass matrix:

$$M_\nu = \frac{4}{c_R}\left[\left(\frac{\kappa}{1+3/c_d}\right)^2 (M_d - M_e) -2\left(\frac{\kappa}{1+3/c_d}\right) M_u + M_u (M_d - M_e)^{-1} M_u\right]$$

- The 2nd line in the above gives almost diagonal entries. So, the large flavor mixings must come from the 1st line in the above.

$$M_\nu|_{\text{flavor mixing}} \propto M_d - M_e$$

- Small (3, 3) element as a result of ***bottom-tau unification is a key point to produce a large mixing angle*** in the neutrino sector.

Bajc, Senjanovic and Vissani, PRL.90:051802 (2003)

$$M_\nu \sim \begin{pmatrix} \sim m_s & \sim m_s \\ \sim m_s & m_b - m_\tau \end{pmatrix} \rightarrow \tan^2 2\theta_{\text{atm}} = \frac{2m_s}{m_b - m_\tau}$$

Bi-large Mixing Mass Matrix

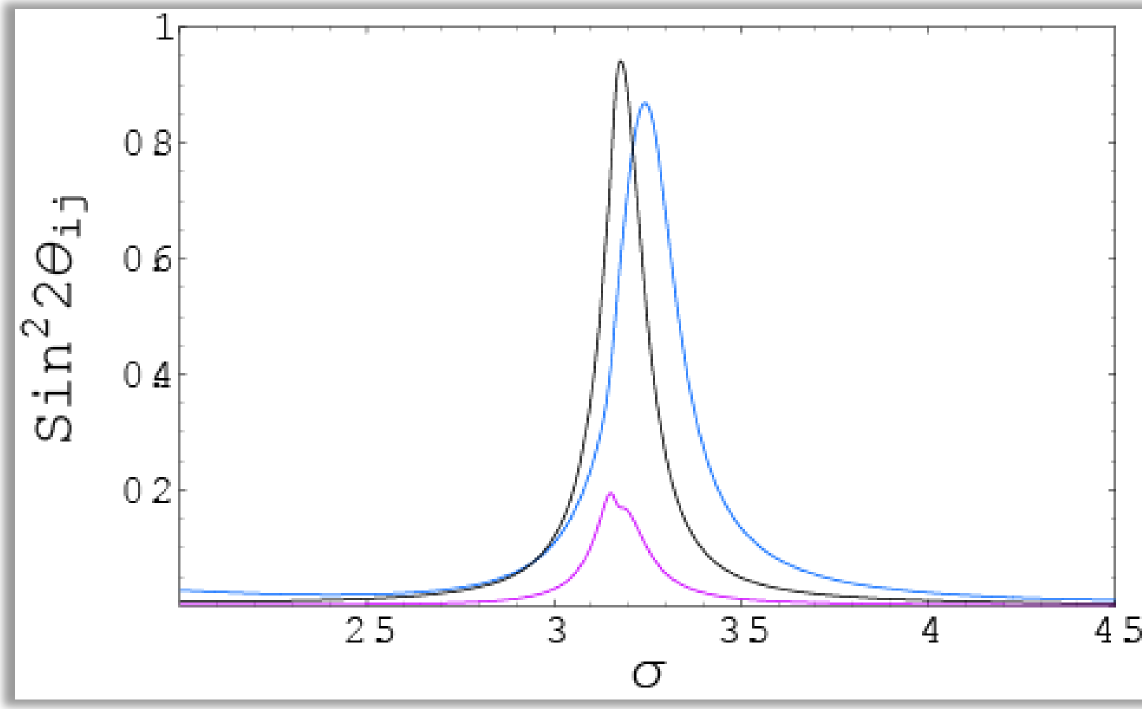
- When we choose parameters to cancel the second line, this relation reduces to the type-II seesaw formula.

$$M_\nu^{\text{type-II}} = (1 - c_d) \times \begin{pmatrix} m_d + V_{us}^2 m_s & V_{us} m_s & V_{ub} m_b \\ V_{us} m_s & m_s + \frac{c_d \kappa}{1-c_d} m_c & V_{cb} m_b \\ V_{ub} m_b & V_{cb} m_b & \frac{m_b + e^{i\sigma} m_\tau}{1-c_d} \end{pmatrix} \sim \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} V_{cb} m_b$$

- **For type-II seesaw form of the mass matrix, large mixings can naturally be obtained as a result of bottom-tau mass unification at the GUT scale.**

Previous analysis of the neutrino mixing angles

- We have only one parameter $\sigma = \arg(c_d)$ left free. So, we can make definite predictions.



$\arg(c_d)$

Fukuyama-Okada ('01)

For $\sigma = 3.198$ [rad] :

$\sin^2 2\theta_{12} \sim 0.72, \quad \sin^2 2\theta_{23} \sim 0.90, \quad \sin^2 2\theta_{13} \sim 0.16$

Updated numerical analysis in detail

- We make use of the following χ^2 function in our data fitting.

$$\chi^2 = \sum_{\alpha=u,\mathrm{c},\mathrm{t},\mathrm{d},\mathrm{s},\mathrm{b},\mathrm{e},\mu,\tau} \left(\frac{m_\alpha - m_\alpha^{mean}}{\Delta m_\alpha} \right)^2$$

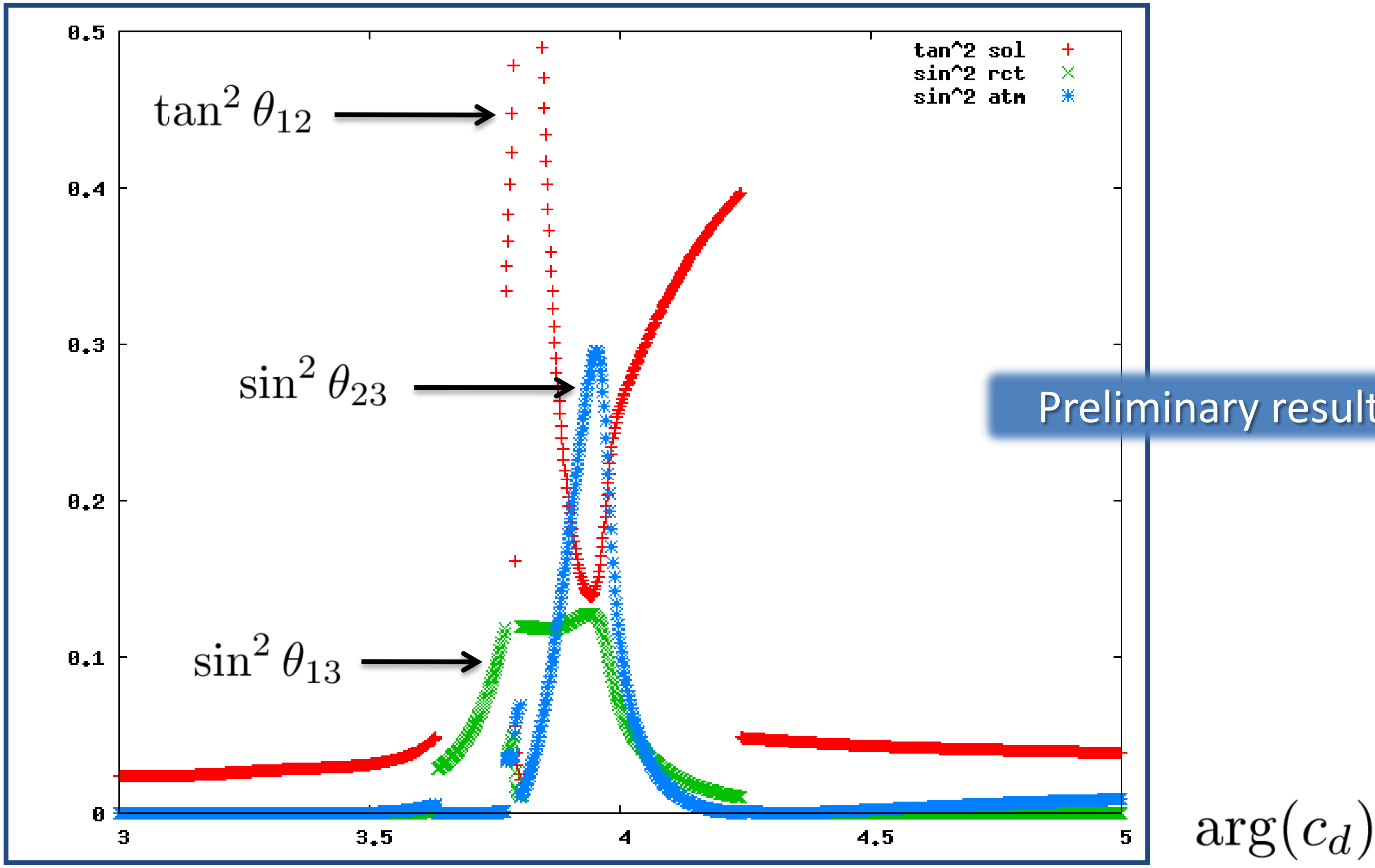
$$+ \sum_{ij=12,13,23} \left(\frac{s_{ij}^{CKM} - s_{ij}^{CKM:mean}}{\Delta s_{ij}^{CKM}} \right)^2 + \left(\frac{\delta^{KM} - \delta^{KM:mean}}{\Delta \delta^{KM}} \right)^2$$

$$+ \left(\frac{\tan^2\theta_{SOL} - \tan^2\theta_{SOL}{}^{mean}}{\Delta \tan^2\theta_{SOL}} \right)^2 + \left(\frac{\sin^2\theta_{ATM} - \sin^2\theta_{ATM}{}^{mean}}{\Delta \sin^2\theta_{SOL}} \right)^2$$

$$+ \left[\frac{\left(\delta m_{SOL}^2 / \delta m_{ATM}^2\right) - \left(\delta m_{SOL}^2 / \delta m_{ATM}^2\right)^{mean}}{\Delta\left(\delta m_{SOL}^2 / \delta m_{ATM}^2\right)} \right]$$

Result of the neutrino mixing angles

<u>Two large mixing angles</u> and <u>one small mixing angle</u> can be obtained.



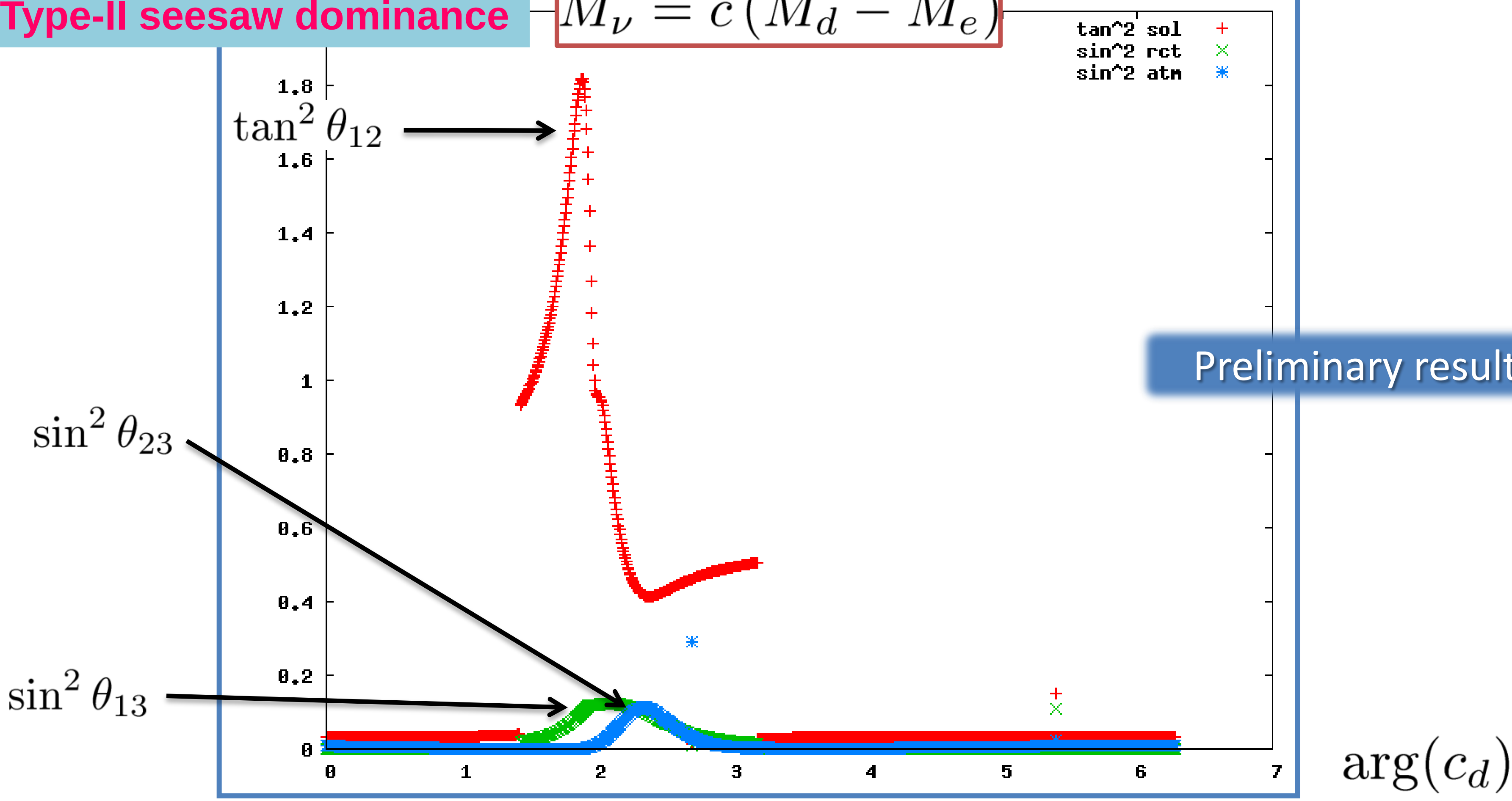
Preliminary result

Understanding the origin of flavor mixing

For comparison, we show the result of mixing angles originated from type-II seesaw.

Type-II seesaw dominance

$$M_\nu = c\,(M_d - M_e)$$

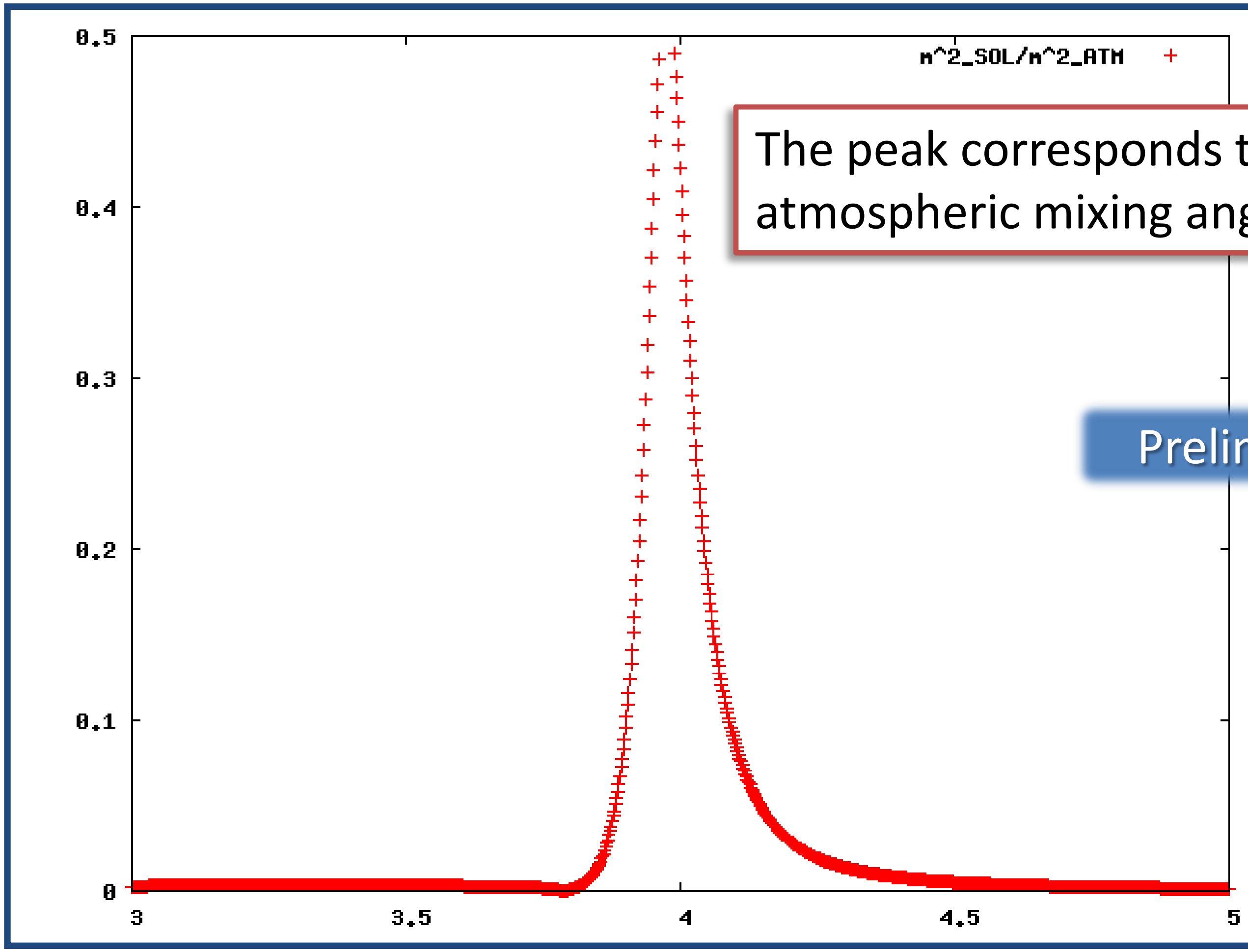


Preliminary result

Result of the mass squared ratio

A bit large mass squared ratio is preferred to get a large mixing angle.

$$\frac{\Delta m^2_{12}}{\Delta m^2_{23}}$$



$$\arg(c_d)$$

The peak corresponds to the large atmospheric mixing angle, $\sin^2\theta_{23}$

Preliminary result

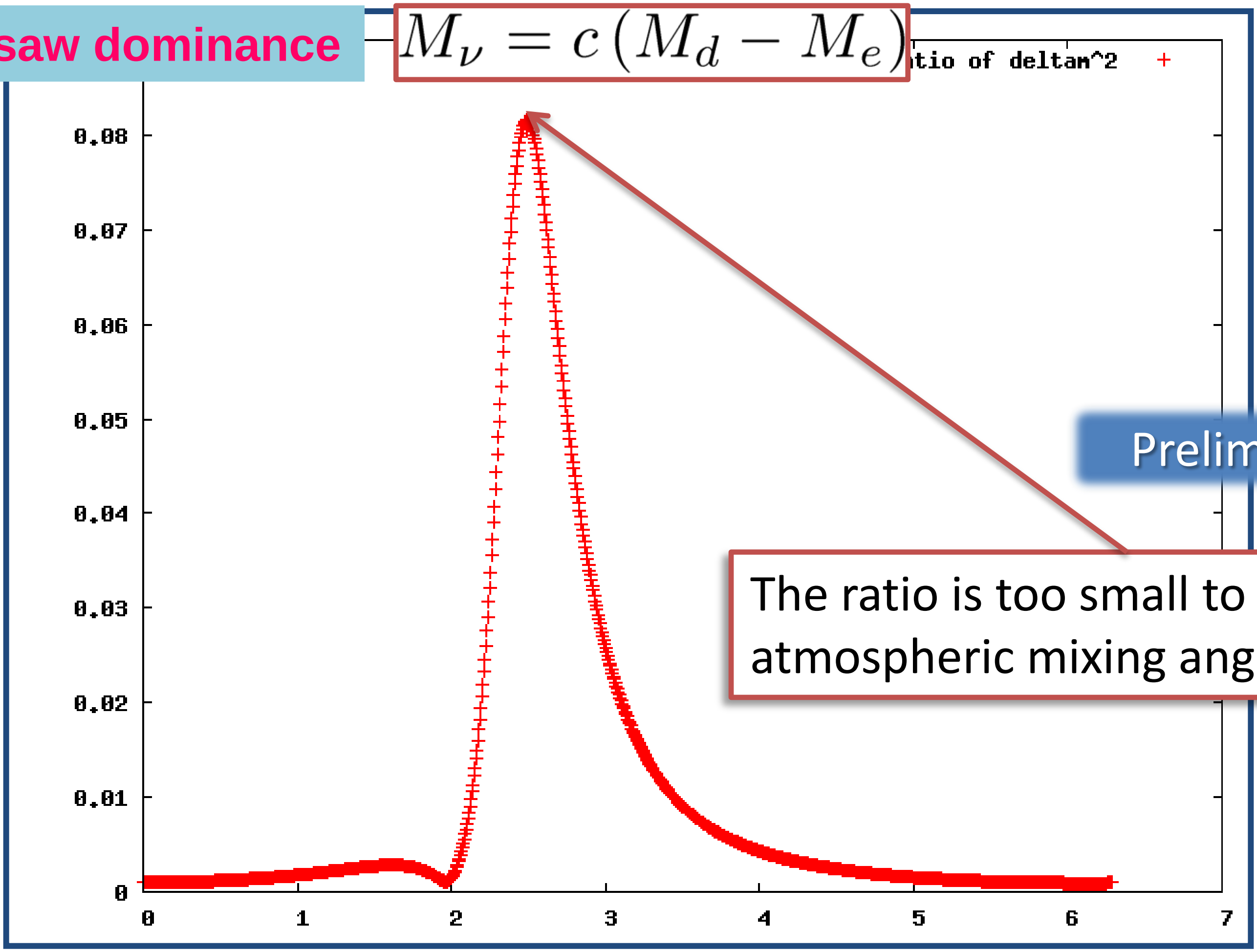
Understanding the origin of mass hierarchy

For comparison, we show the result of mass ratio originated from type-II seesaw.

Type-II seesaw dominance

$$M_\nu = c\,(M_d - M_e)$$

$$\frac{\Delta m^2_{12}}{\Delta m^2_{23}}$$

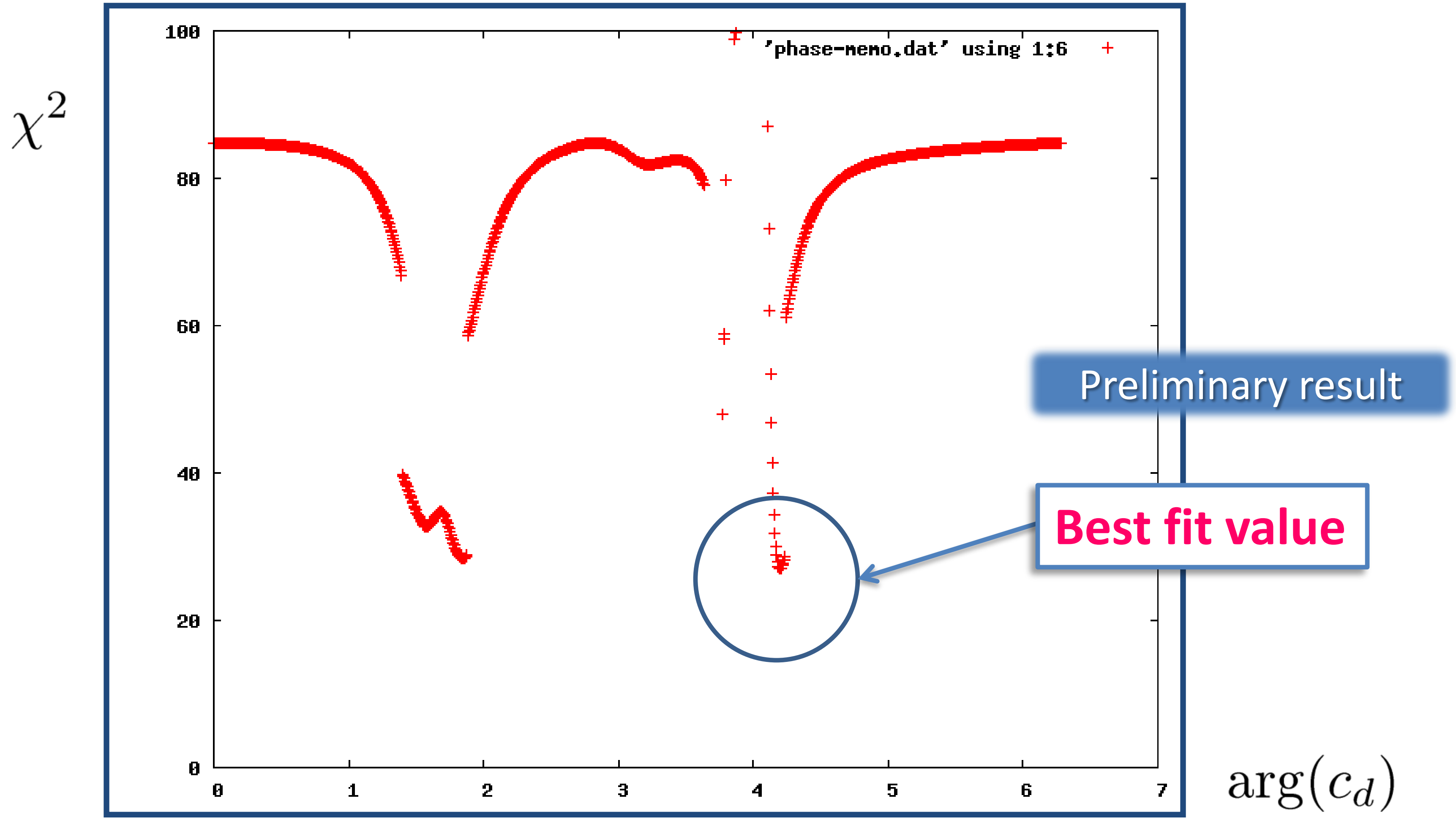


$\arg(c_d)$

Preliminary result

The ratio is too small to have large atmospheric mixing angle, $\sin^2\theta_{23}$

Goodness of the fit -χ^2 function



Best fit values - χ^2_{min}

Preliminary result

Charged fermion data can be fitted well

$$m_u = -0.492 \text{ [MeV]}$$
$$m_c = +0.234 \text{ [GeV]}$$
$$m_t = -90.16 \text{ [GeV]}$$
$$m_d = -0.559 \text{ [MeV]}$$
$$m_s = -16.60 \text{ [MeV]}$$
$$m_b = +0.789 \text{ [GeV]}$$
$$\sin\theta_{12} = 0.226$$
$$\sin\theta_{13} = 0.0392$$
$$\sin\theta_{23} = 0.0369$$
$$\delta_{\rm CKM} = -1.07 \text{ [rad]}$$

GUT parameters can be fixed

$$|\kappa| = 0.01002$$
$$\arg(\kappa) = 0.1396 \text{ [rad]}$$
$$|c_d| = 5.626$$
$$\arg(c_d) = 4.197 \text{ [rad]}$$

Predicted values on the neutrino oscillation parameters

$$\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm} = 0.00142$$
$$\tan^2\theta_{\rm sol} = 0.0299$$
$$\sin^2\theta_{\rm atm} = 0.3810$$
$$\sin^2\theta_{13} = 0.0131$$

Summary

- We have re-evaluated neutrino oscillation data in the minimal SO(10) model using the updated values of experimental data.

$$\mathcal{L}_{\text{Yukawa}} = f_{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H + h_{ij} \mathbf{16}_i \mathbf{16}_j \overline{\mathbf{126}}_H$$

$$M_e = c_d \left(M_d + \kappa M_u \right)$$

$$M_\nu = -M_D M_R^{-1} M_D$$

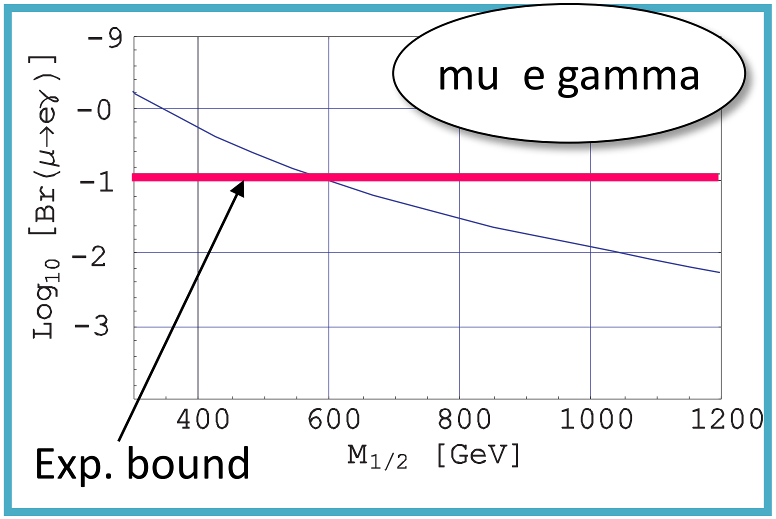
$$M_D = M_u + \frac{1 + 3/c_d}{\kappa} \left(M_d - M_e \right)$$

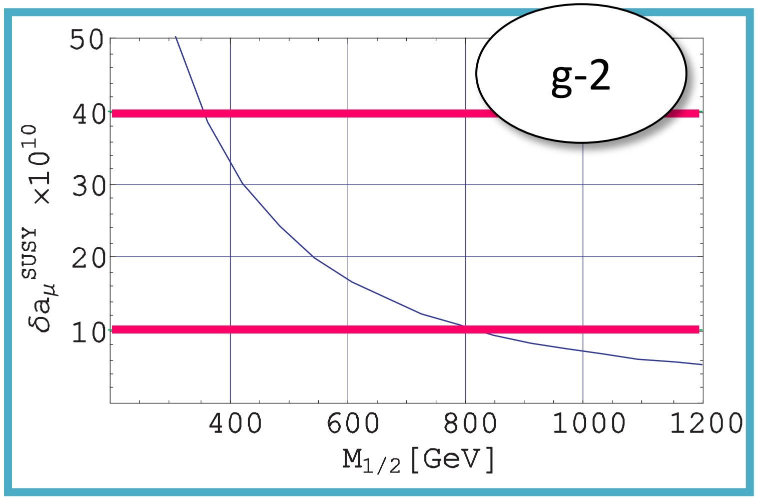
$$M_R = c_R \left(M_d - M_e \right)$$

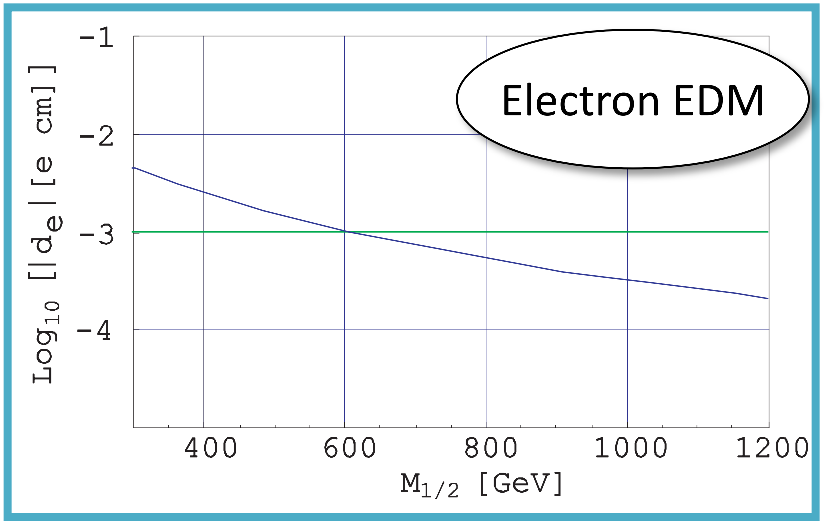
- *The model has less parameters (only 11 real + some phases).*
- *Using well-established quark and charged-lepton data (13 data),* ***we can give a definite prediction on the neutrino parameters!***
- Results
  - ✓ ***All the masses and CKM mixing matrix can be fitted.***
  - ✓ ***Neutrino masses & mixings can be well much described.***

BACKUP SLIDES

Other predictions on LFV, g-2, EDM

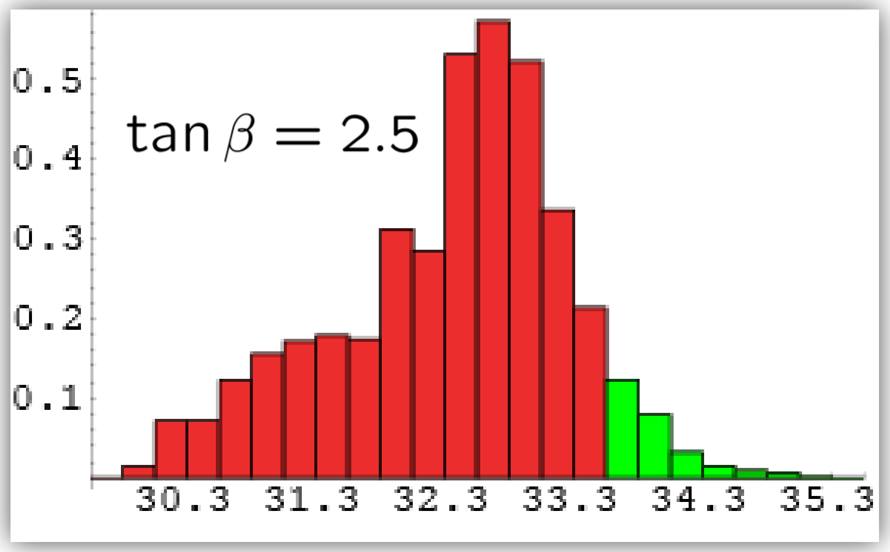


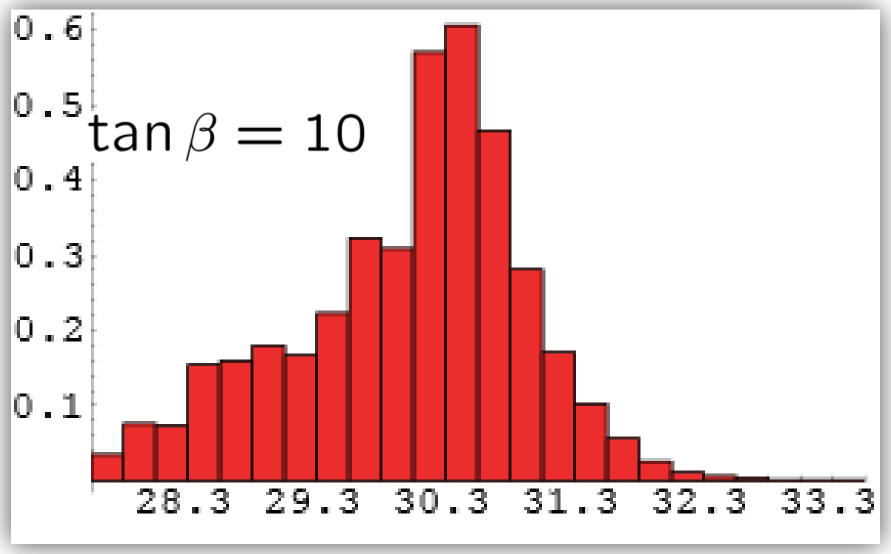




Fukuyama-T.K.-Okada ('03)

Proton decay rate in minimal SO(10) model





$\tau(p \to K^+\bar{\nu}) \propto \tan^2\beta$

Fukuyama-Ilakovac-T.K.-Meljanac-Okada ('05)

The green region is the allowed region.

In case of $\tan\beta = 2.5$, there is a large enough parameter space where all the proton decay models are canceled out although it is very tiny region in case of $\tan\beta = 10$

References

My previous SO(10) works in collaboration with
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