Electroweak phase transition in the light Higgs boson scenario

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Outline

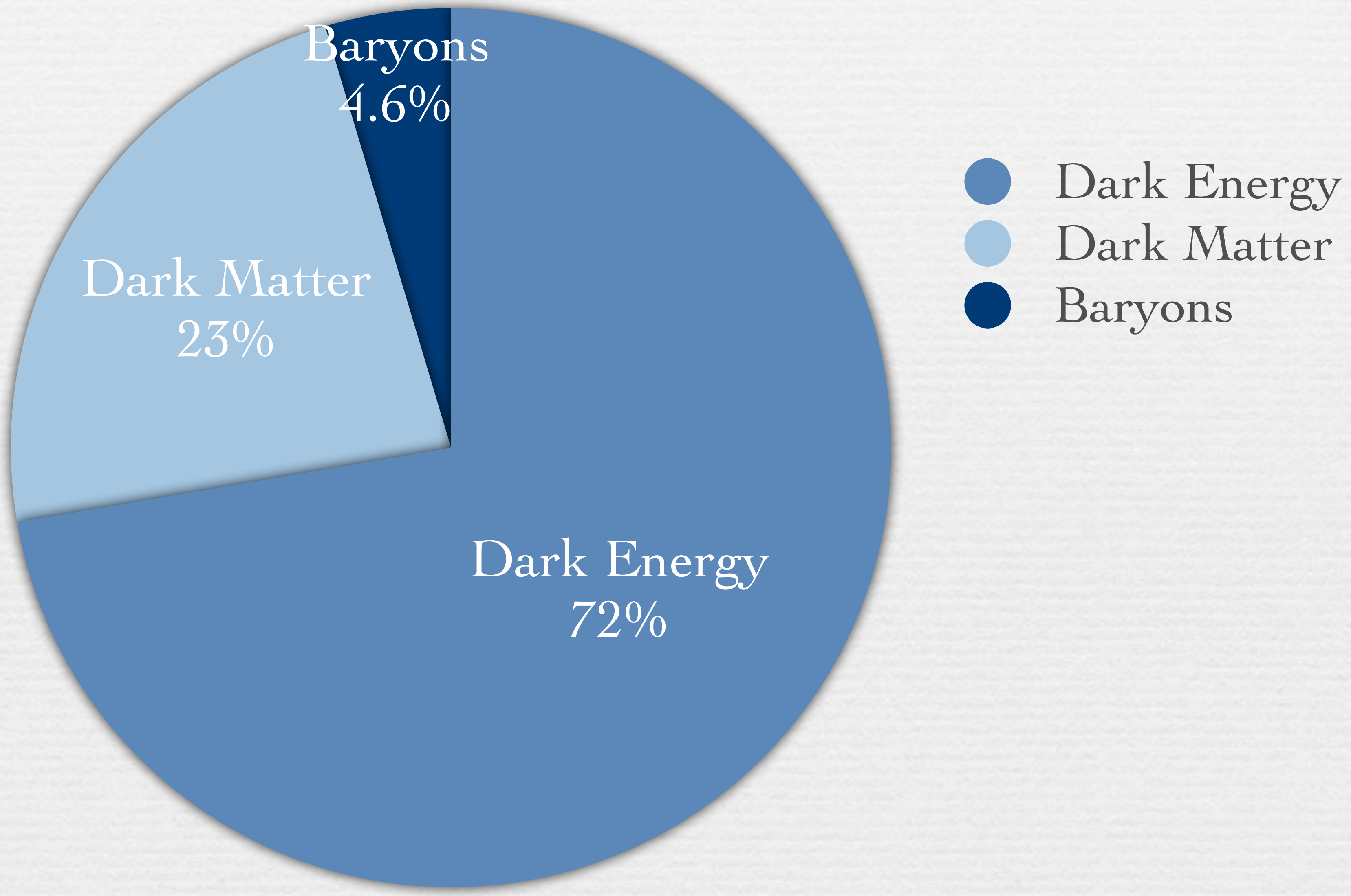
- Motivation
  Baryon Asymmetry of the Universe (BAU)

- Electroweak baryogenesis in the light Higgs boson scenario of the MSSM
  - Sphaleron decoupling condition
  - Electroweak phase transition (EWPT)
  - Critical bubble

- Summary

Motivation

- Energy budget of the Universe



- 95% of the Universe is made of dark object.
- It should be stressed that there remains a mystery in the visible sector as well.

Where did antibaryons go?

Motivation

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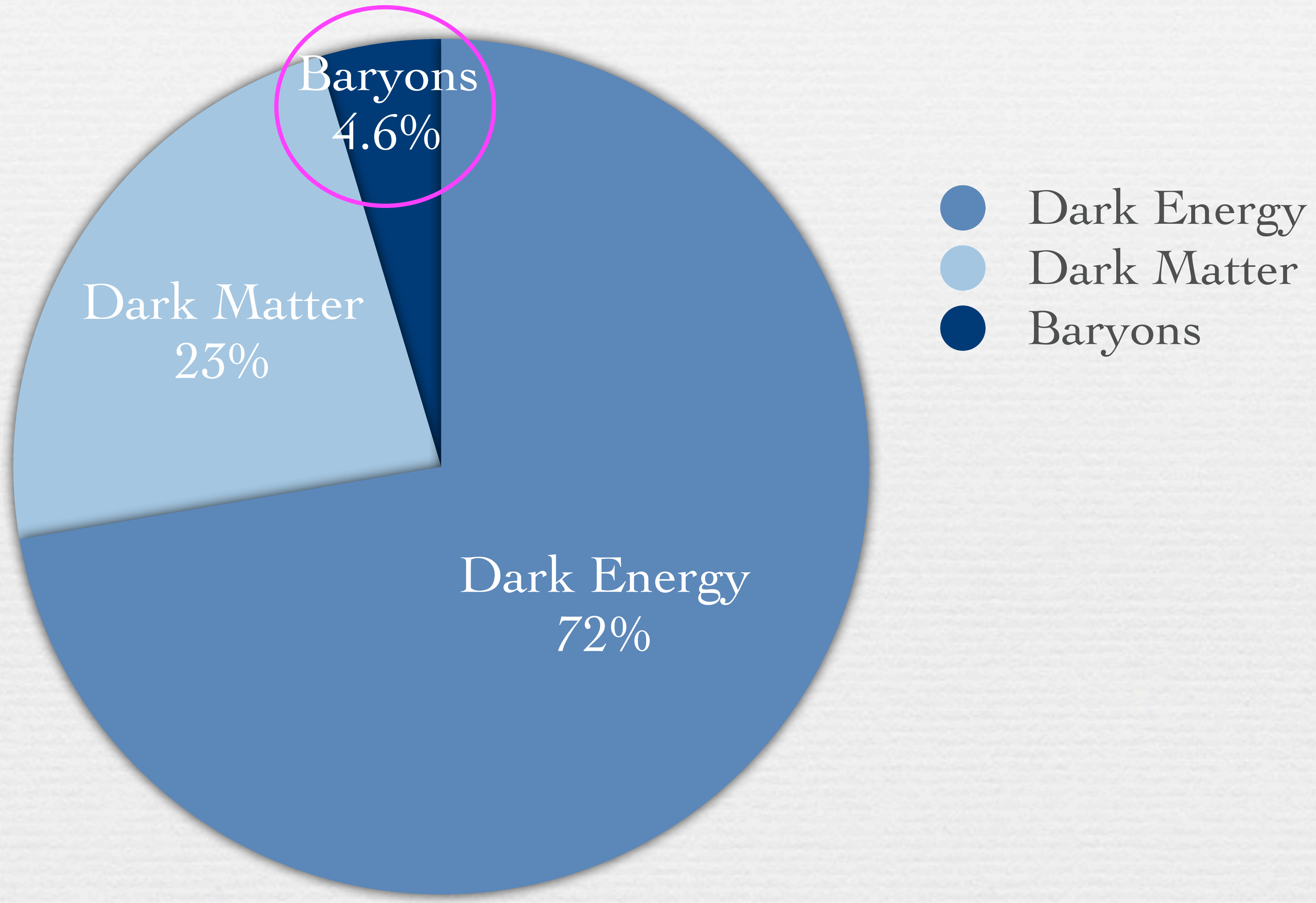

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Where did antibaryons go?

BAU

❧ Baryon Asymmetry of the Universe (BAU)

$$\frac{n_B}{n_\gamma} = \frac{n_b - n_{\bar{b}}}{n_\gamma} = (4.7 - 6.5) \times 10^{-10} \quad (95\% \text{ C.L.})$$

(PDG 08)

Sakharov's conditions ('67)	SM case
(1) Baryon(B) violation	○ sphaleron process
(2) C violation	○ chiral gauge interaction
CP violation	× KM phase is not sufficient
(3) out of equilibrium	× phase transition is not 1st order

Extensions of the SM

MSSM, 2HDM, singlet-extended MSSM etc.

Today, we discuss the MSSM baryogenesis (BG).

Tension in the MSSM BG

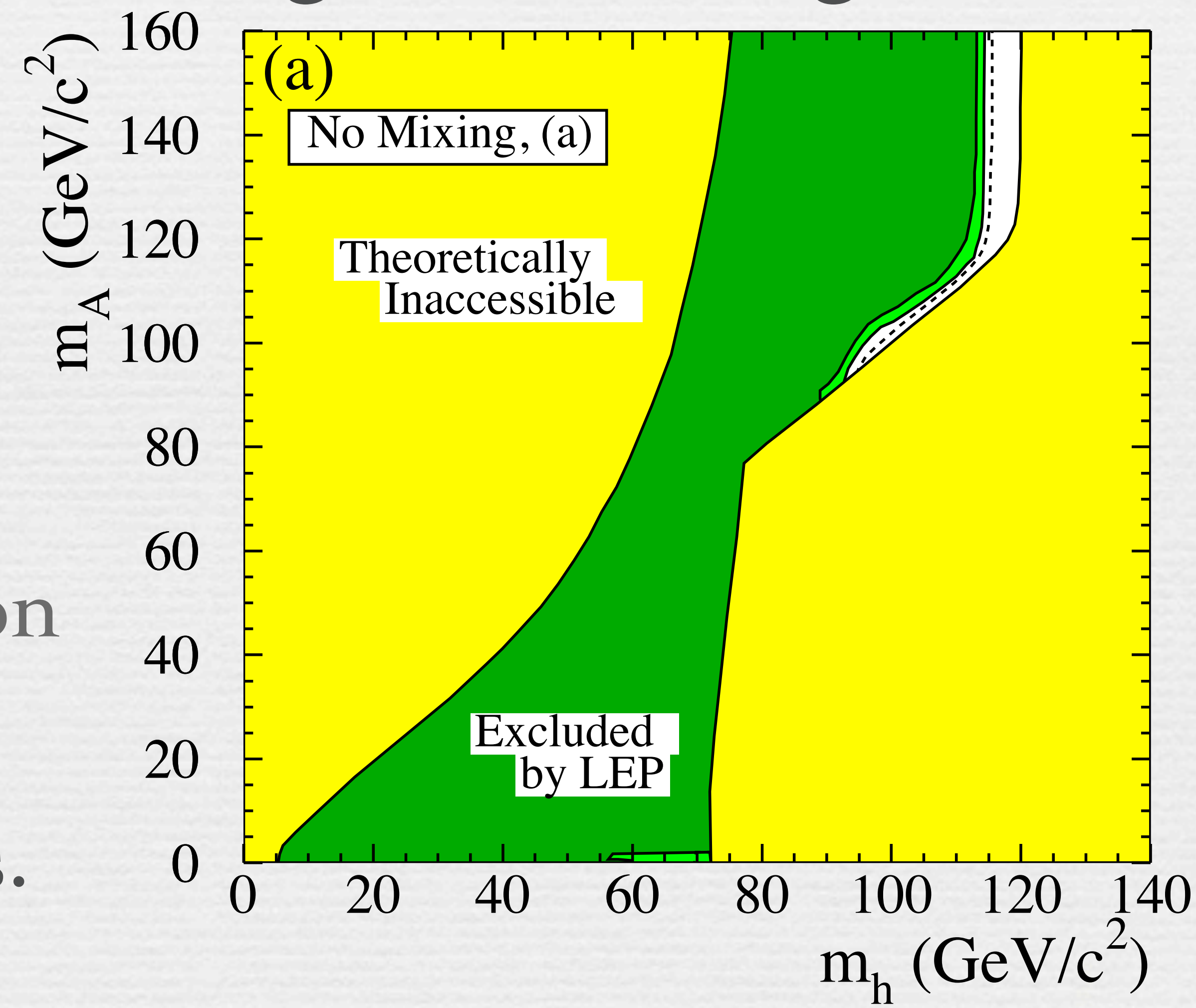
- From the EWPT point of view, the light Higgs boson is generically favored.

- The LEP data put a strong constraint on the light Higgs boson.

On the other hand,

- In the literatures, $\frac{v_C}{T_C} \gtrsim 1$ is used as the practical criterion for the strong 1st order PT.

e.g. no-mixing case



But,

- v_c/T_c>0.9 region or v_c/T_c>1.1 region are completely different.

⇒ Need to reduce the uncertainties.

- We analyze the EWPT in the light Higgs boson scenario (m_h<114.4 GeV) based on the refined sphaleron decoupling cond.

Effective potential

- To discuss the PT we use the effective potential.
- The gauge bosons and 3rd generation of quarks/squarks are taken into account.

$$V_{\rm eff}(\Phi_d,\Phi_u)=V_0(\Phi_d,\Phi_u)+\Delta V(\Phi_d,\Phi_u;T),$$

Tree:
$$\begin{aligned}V_0(\Phi_d,\Phi_u) &= m_1^2\Phi_d^\dagger\Phi_d+m_2^2\Phi_u^\dagger\Phi_u-(m_3^2\epsilon_{ij}\Phi_d^i\Phi_u^j+\text{h.c.})\\ &\quad +\frac{g_2^2+g_1^2}{8}(\Phi_d^\dagger\Phi_d-\Phi_u^\dagger\Phi_u)^2+\frac{g_2^2}{2}(\Phi_d^\dagger\Phi_u)(\Phi_u^\dagger\Phi_d),\end{aligned}$$

1-loop:
$$\Delta V(\Phi_d,\Phi_u;T)=\sum_A c_A\left[F_0(\bar m_A^2)+\frac{T^4}{2\pi^2}I_{B,F}\left(\frac{\bar m_A^2}{T^2}\right)\right]$$

$$F_0(m^2)=\frac{m^4}{64\pi^2}\left(\ln\frac{m^2}{M^2}-\frac{3}{2}\right),\quad I_{B,F}(a^2)=\int_0^\infty dx\ x^2\ln\left(1\mp e^{-\sqrt{x^2+a^2}}\right)$$

Fitting function:

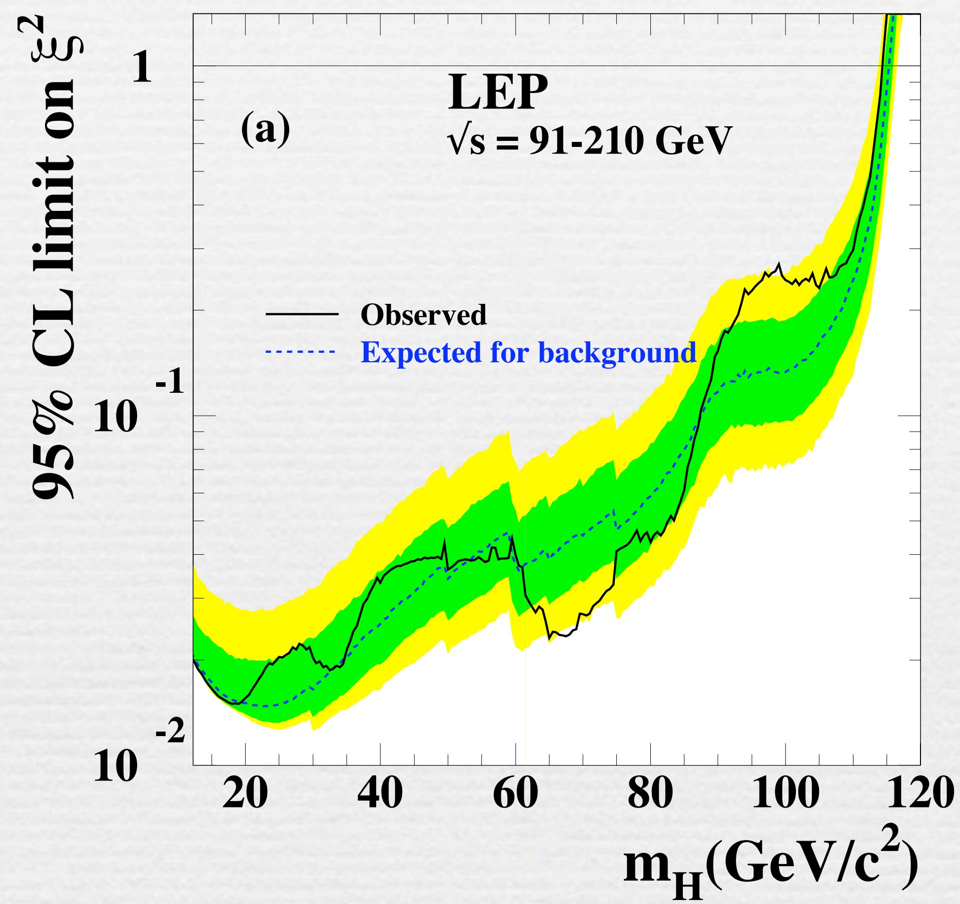
$$\tilde I_{B,F}(a^2)=e^{-a}\sum_{n=0}^{N}c_n^{b,f}a^n,\quad |\tilde I_{B,F}-I_{B,F}|<10^{-6}\ (N=40).$$

- The fitting function is used in our numerical analysis.

Light Higgs boson scenario (LHS)

$$m_h < 114.4 \text{ GeV}$$

LEP data $\xi = \frac{g_{\phi ZZ}}{g_{h^{\rm SM}ZZ}}$



Higgs fields in the rotated basis:

$$(i\tau^2\Phi_d'^*\ \Phi_u')^T = O(\beta)(i\tau^2\Phi_d^*\ \Phi_u)^T$$

$$\Phi_d' = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_0 + h_d' + iG^0) \\ G^- \end{pmatrix}, \quad \Phi_u' = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h_u' + iA) \end{pmatrix}$$

where $h_d' = h^{\rm SM}$

$$\begin{pmatrix} h_d' \\ h_u' \end{pmatrix} = \begin{pmatrix} \cos(\beta-\alpha) & \sin(\beta-\alpha) \\ -\sin(\beta-\alpha) & \cos(\beta-\alpha) \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}$$

Higgs couplings:

$$\frac{g_{hZZ}}{g_{h^{\rm SM}ZZ}} = \sin(\beta-\alpha), \quad \frac{g_{HZZ}}{g_{h^{\rm SM}ZZ}} = \cos(\beta-\alpha),$$
$$\frac{g_{hZA}}{g_{h^{\rm SM}ZG^0}} = \cos(\beta-\alpha), \quad \frac{g_{HZA}}{g_{h^{\rm SM}ZG^0}} = \sin(\beta-\alpha).$$

LHS: $\cos(\beta-\alpha) \to 1,\ H \to h^{\rm SM},\ m_A \sim m_Z$

Decoupling lim: $\sin(\beta-\alpha) \to 1,\ h \to h^{\rm SM},\ \ m_A \gg m_Z.$

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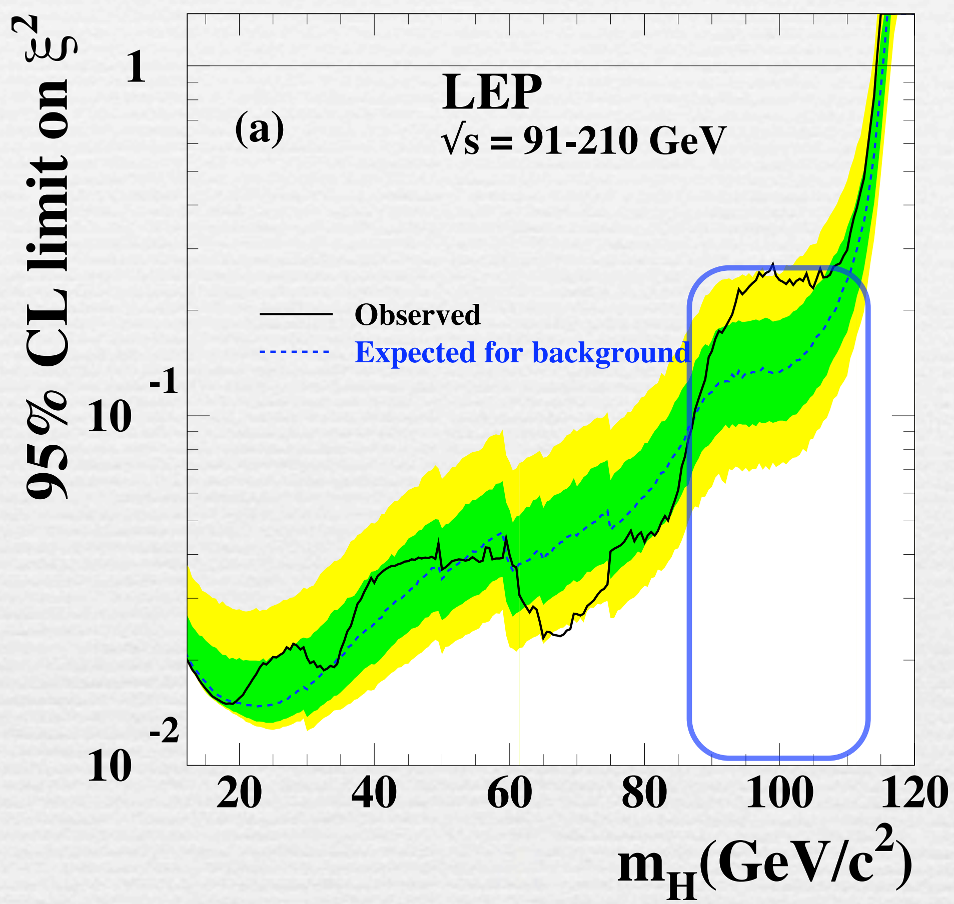

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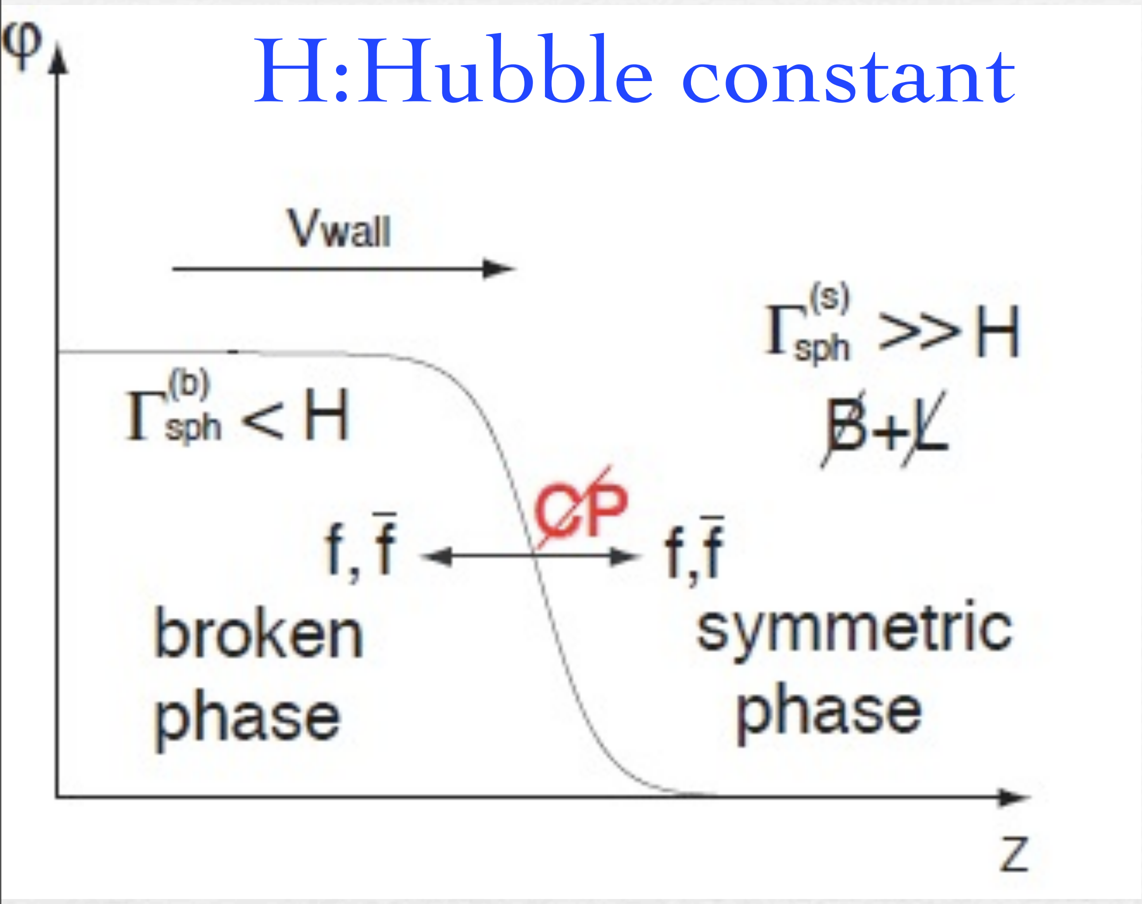
Mechanism of EWBG

[Kuzmin, Rubakov, Shaposhnikov, PLB155,36 ('85)]

- The EWPT must be a first order with expanding bubble walls.



outline

- Due to *CP* violation at the phase boundary, asymmetries of particle number densities occur.
- They diffuse into symmetric phase.
- Left-handed particle number densities are converted into B via sphaleron process.
- Sphaleron process is decoupled after the PT.
- B is frozen.

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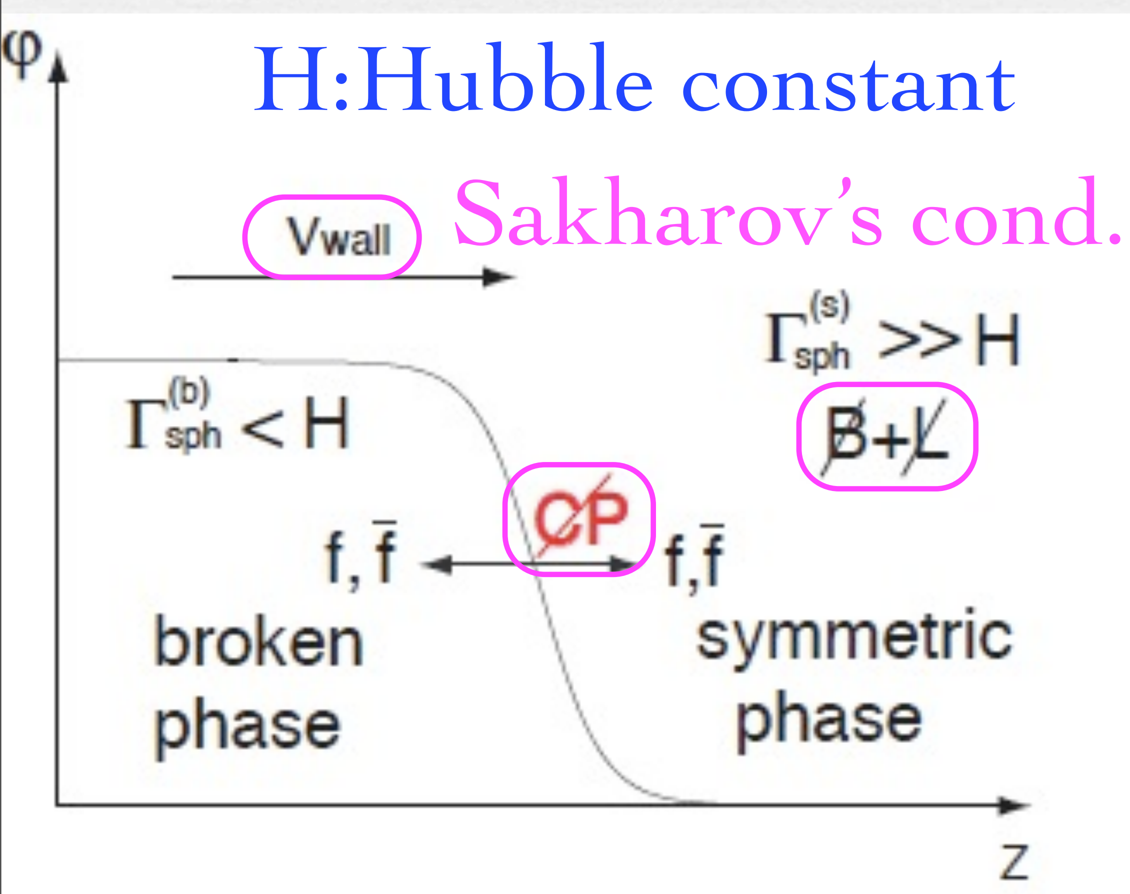

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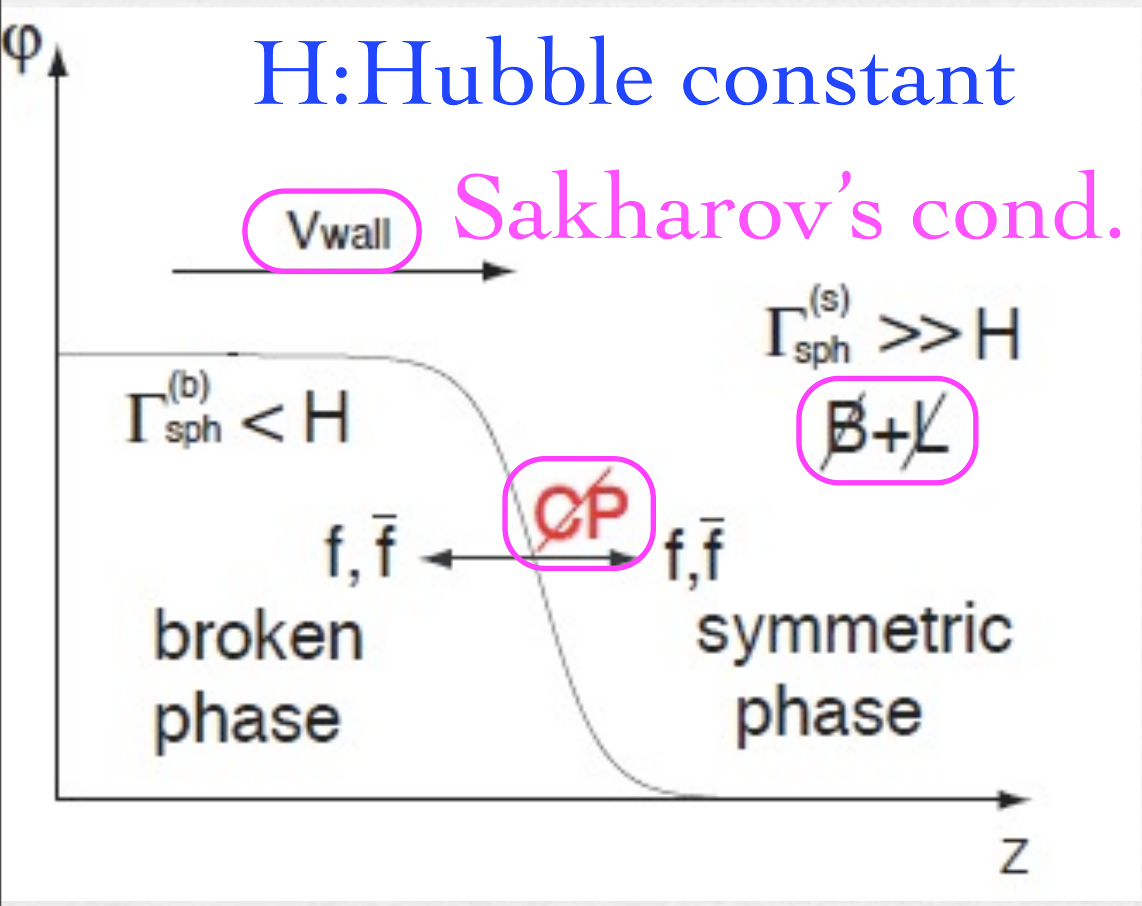
H:Hubble constant

Sakharov's cond.



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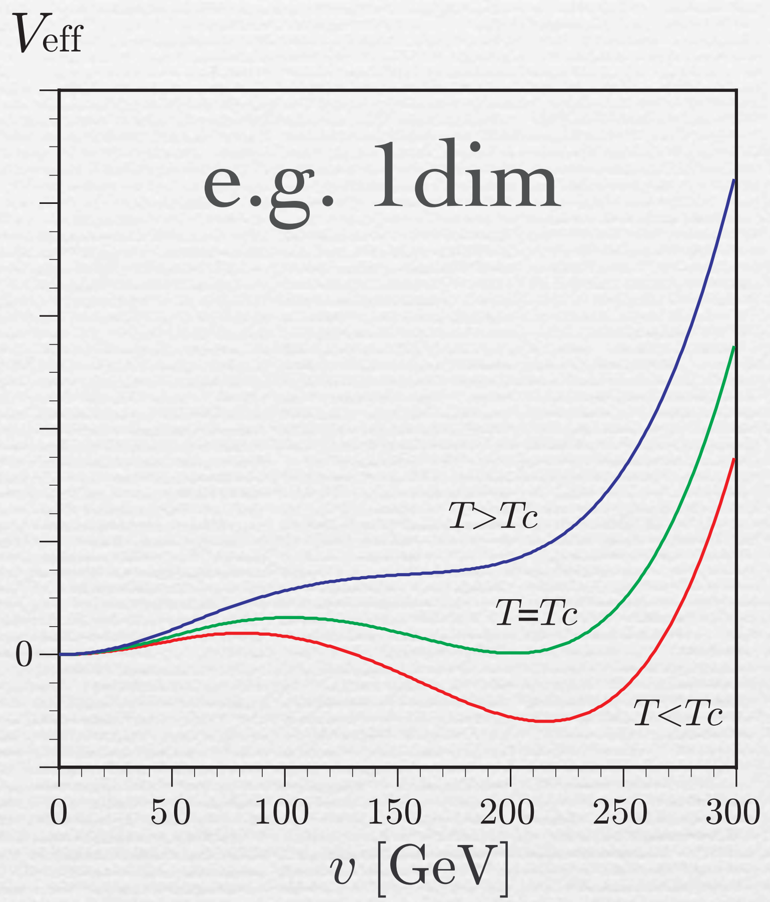
Phase transition

- The PT must be 1st order to realize out of equilibrium.



- order parameters = Higgs VEVs
- At T_c, the potential has two degenerate minima.

- Light stop plays a crucial role in strengthening the 1st order PT.

[Carena, Quiros, Wagner, PLB380 ('96) 81]

To be consistent with the LEP bound on m_H and ρ-parameter, we should take

$$m_{\tilde{q}}^2 \gg m_{\tilde{t}_R}^2, X_t^2, \quad X_t = A_t - \mu \cot\beta$$

lighter stop mass $\Rightarrow \bar{m}_{\tilde{t}_1}^2 = m_{\tilde{t}_R}^2 + \frac{y_t^2 \sin^2\beta}{2}\left(1 - \frac{X_t^2}{m_{\tilde{q}}^2}\right) v^2 + \mathcal{O}(g^2)$

c.f. High T expansion:

$$V_{\rm eff} \ni -(E_{\rm SM} + E_{\tilde{t}_1}) T v^3 \quad E_{\tilde{t}_1} \simeq \frac{y_t^3 \sin^3\beta}{4\sqrt{2}\pi}\left(1 - \frac{X_t^2}{m_{\tilde{q}}^2}\right)^{3/2}$$

~ 0.01 ~ 0.06 (Xt=0)

large effect!

Phase transition

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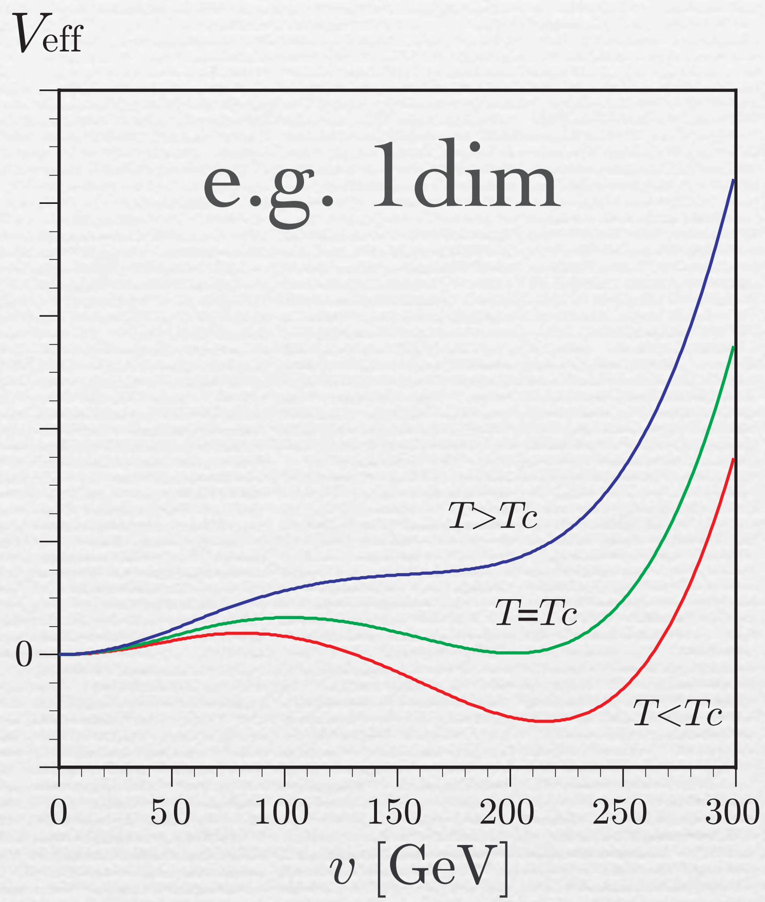

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$m_{\tilde{t}_R}^2 \rightarrow 0 \; (m_{\tilde{t}_1} < m_t)$

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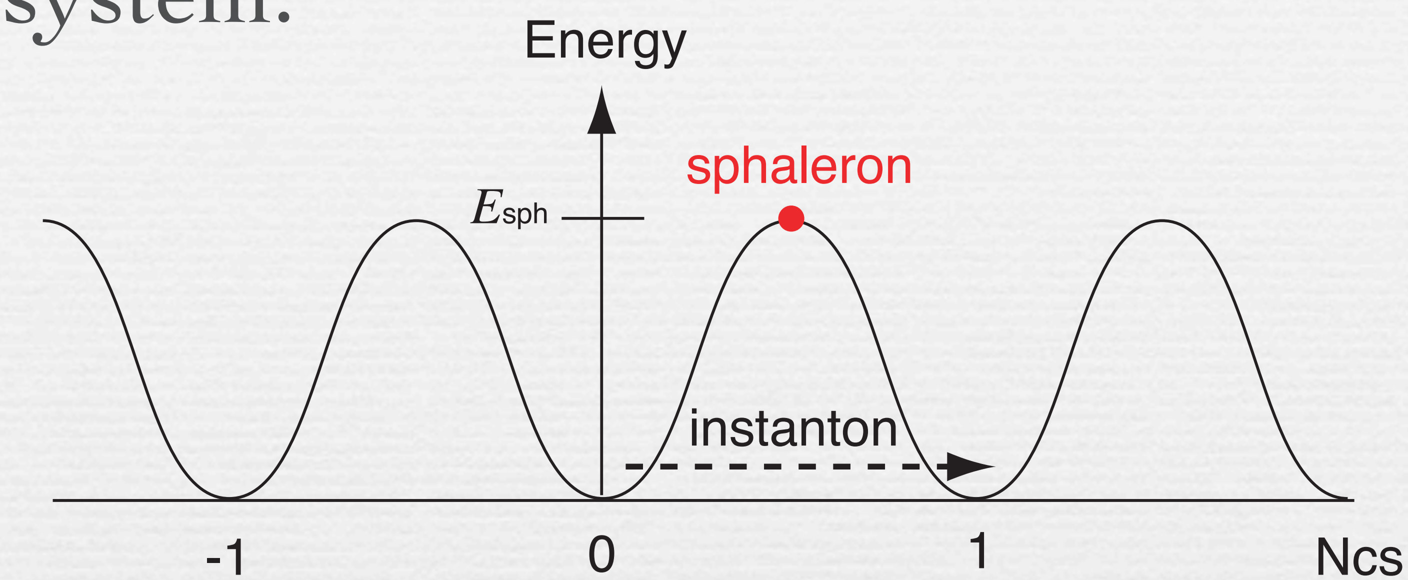
$E_{\rm SM} \sim 0.01$

$E_{\tilde{t}_1} \sim 0.06$ (Xt=0) large effect!

Sphaleron

- A static saddle point solution w/ finite energy of the gauge-Higgs system. [N.S. Manton, PRD28 ('83) 2019]



$\underline{\Delta B \neq 0}$

Instanton: quantum tunneling

Sphaleron: thermal fluctuation

B violation:

$$\Delta B = 3\Delta N_{CS} \qquad N_{CS} = \frac{g_2^2}{32\pi^2}\int d^3x\ \epsilon_{ijk}\mathrm{Tr}\left[F_{ij}A_k - \frac{2}{3}g_2 A_i A_j A_k\right]$$

Vacuum transition rates:

In the symmetric phase : $\sim \kappa(\alpha_W T)^4, \quad \alpha_W = g_2^2/(4\pi)$

In the broken phase : $\sim T^3 e^{-E_{\text{sph}}/T}$.

At $T = 0$: $\sim e^{-S_{\text{instanton}}} = e^{-8\pi^2/g_2^2} \ll 1$ no proton decay

- B violating process is active at finite T but is suppressed at T=0.

Sphaleron decoupling condition

▪ To avoid the washout of the generated BAU, the sphaleron process must be decoupled after the PT.

Hubble constant

$$-\frac{1}{B}\frac{dB}{dt} \simeq \frac{13\cdot 3}{4\cdot 32\pi^2}\frac{\omega_-}{\alpha_W^3}\kappa\mathcal{N}_{\rm tr}\mathcal{N}_{\rm rot}e^{-E_{\rm sph}/T} < H(T) \simeq 1.66\sqrt{g_*}T^2/m_{\rm P}$$

$E_{\rm sph}$: sphaleron energy, $\mathcal{N}_{\rm tr}, \mathcal{N}_{\rm rot}$: zero mode factors

ω_- : negative mode, $\kappa = \mathcal{O}(1)$.

If we denote $E_{\rm sph} = 4\pi v\mathcal{E}/g_2$

$$\frac{v}{T} > \frac{g_2}{4\pi\mathcal{E}}\left[42.97 + \ln(\kappa\mathcal{N}_{\rm tr}\mathcal{N}_{\rm rot}) + \ln\left(\frac{\omega_-}{m_W}\right) - \frac{1}{2}\ln\left(\frac{g_*}{106.75}\right) - 2\ln\left(\frac{T}{100\ \text{GeV}}\right)\right]$$

In the SM:

$$\mathcal{E} = 2.00,\ \mathcal{N}_{\rm tr}\mathcal{N}_{\rm rot} = 80.13,\ \omega_-^2 = 2.3m_W^2,\ \kappa = 1,\ T = 100\ \text{GeV},\ \lambda = g_2^2.$$

$$\frac{v}{T} > 0.026 \times (42.97 + 4.38 + 0.416) = 1.24$$

10% correction

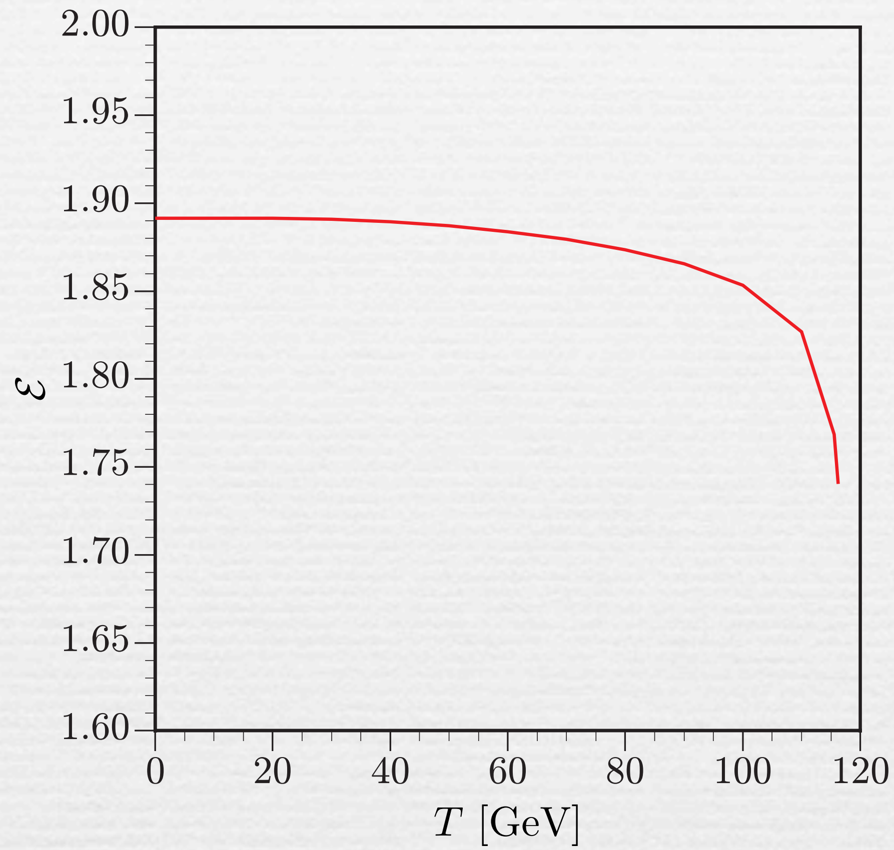
MSSM case

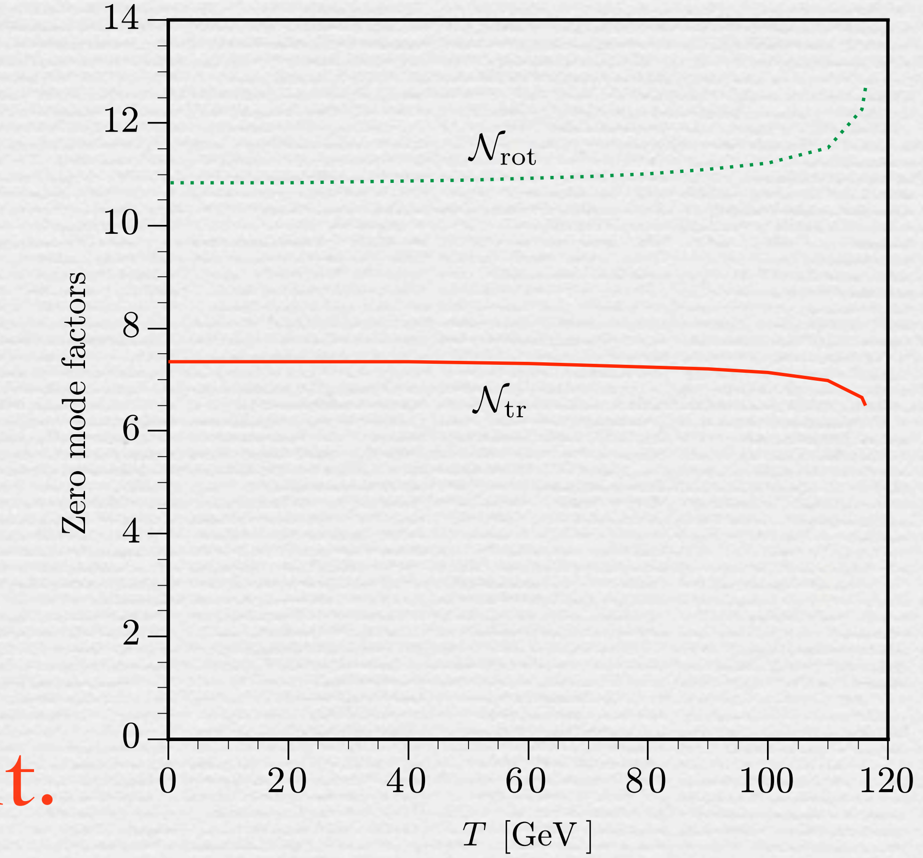
- Effects of T and zero modes

I: based on $V_{\rm eff}(T=0)$ *without* the zero modes
II: based on $V_{\rm eff}(T=0)$ *with* the zero modes
III: based on $V_{\rm eff}(T\neq 0)$ with the zero modes

For the typical parameter set

	I	II	III
$\mathcal{E}$	1.89	1.89	1.77
$\mathcal{N}_{\rm tr}$	—	7.36	6.65
$\mathcal{N}_{\rm rot}$	—	10.84	12.27
$v_N/T_N >$	1.17	1.29	1.38

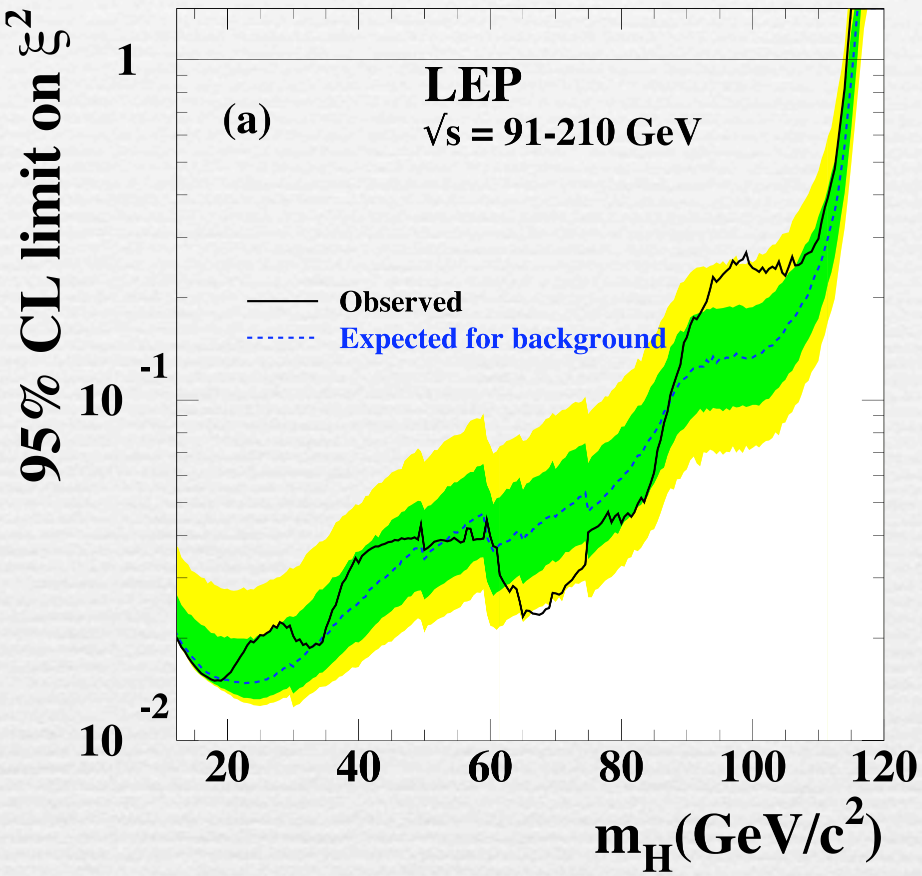




- Zero mode factors cannot be neglected.
- T-dependence must be taken into account.

Experimental constraints

e.g. Higgsstrahlung



- Higgs bounds@ [PLB565, 61 (2003)]

$$g^2_{H_iZZ} \times \mathrm{Br}(H_i \to f\bar{f}) < \mathcal{F}_{H_iZ}(m_{H_i}),$$

$$g^2_{H_iH_jZ} \times \mathrm{Br}(H_i \to f\bar{f}) \times \mathrm{Br}(H_j \to f\bar{f}) < \mathcal{F}_{H_iH_j}(m_{H_i} + m_{H_j}),$$

where $f = b, \tau$. $\mathcal{F}_{H_iZ}$ and $\mathcal{F}_{H_iH_j}$ are the 95% C.L. upper limits

- Lower bounds for SUSY particles:

  e.g. chargino mass > 94 GeV

- ρ-parameter:

$$\Delta\rho \equiv \frac{\Pi^T_{ZZ}(0)}{m_Z^2} - \frac{\Pi^T_{WW}(0)}{m_W^2} < 0.002$$

- B physics observables:

$$\mathrm{Br}(B_u \to \tau\nu_\tau)_{\mathrm{exp}} = 1.41^{+0.43}_{-0.42} \times 10^{-4},$$

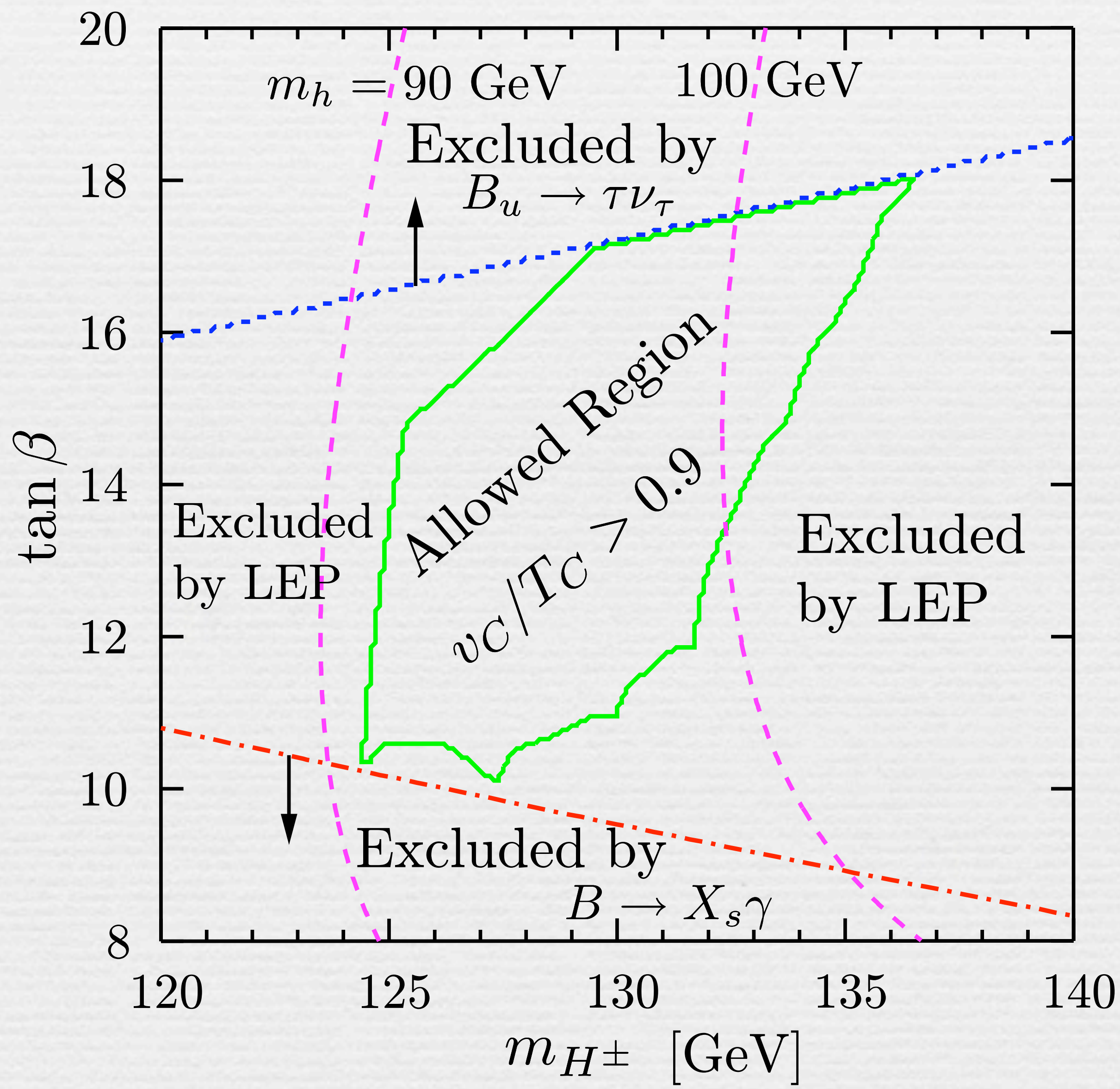
$$\mathrm{Br}(\overline{B} \to X_s\gamma)_{\mathrm{exp}} = (3.52 \pm 0.23 \pm 0.09) \times 10^{-4},$$

$$\mathrm{Br}(B_s \to \mu^+\mu^-)_{\mathrm{exp}} < 0.23 \times 10^{-7}.$$

Allowed region

- The allowed region is highly constrained by the experimental data.

$m_{\tilde{q}} = 1200$ GeV, $m_{\tilde{t}_R} \simeq 0$, $A_t = A_b = -300$ GeV.



Maximal v/T:

$\tan\beta = 10.1,\ m_{H^\pm} = 127.4$ GeV

$$\frac{v_C}{T_C} = \frac{107.10 \text{ GeV}}{116.27 \text{ GeV}} = 0.92$$

↓

The sphaleron process is not decoupled at Tc.

Loophole: supercooling

$\Rightarrow$ The PT begins to proceed with bubble wall at below Tc.

We need to know the critical bubbles.

Critical bubble

- For the EWPT to proceed, the radius of bubble must be larger than some critical size.

Higgs fields:

$$\Phi_d = \frac{1}{\sqrt{2}}\begin{pmatrix} \rho_d \\ 0 \end{pmatrix}, \quad \Phi_u = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \rho_u \end{pmatrix},$$

Energy functional:

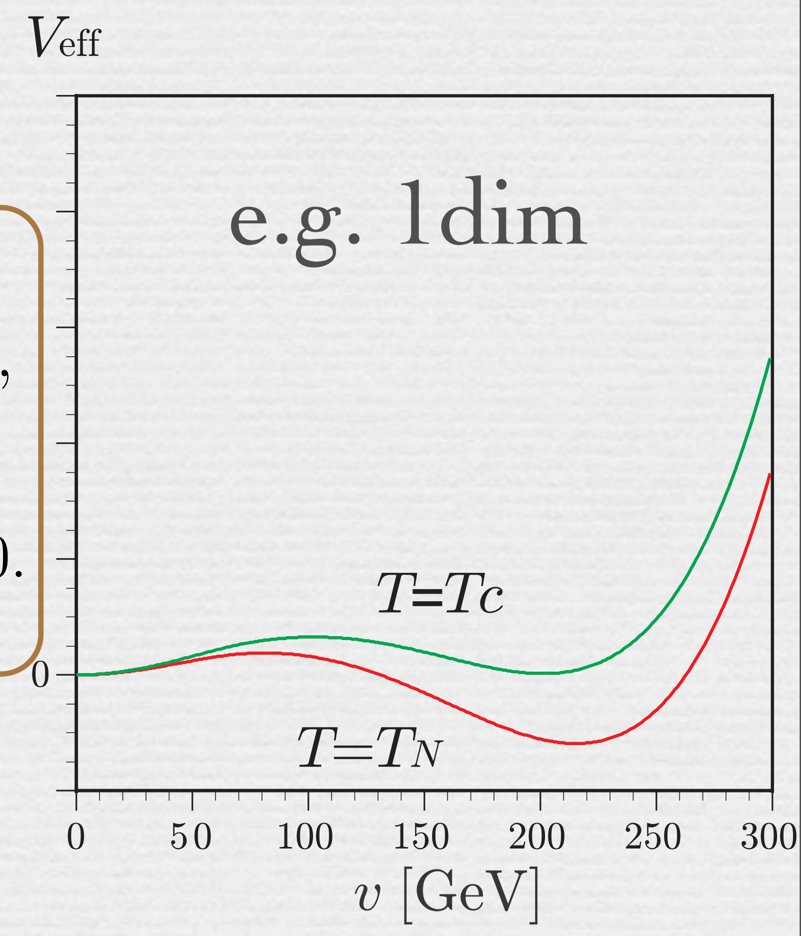
$$E = 4\pi \int_0^\infty dr\, r^2 \left[\frac{1}{2}\left\{\left(\frac{d\rho_d}{dr}\right)^2 + \left(\frac{d\rho_u}{dr}\right)^2\right\} + V_{\rm eff}(\rho_d, \rho_u; T)\right] \quad r = \sqrt{\boldsymbol{x}^2}$$

Equation of motion (EOM):

$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\rho_d}{dr}\right) + \frac{\partial V_{\rm eff}}{\partial \rho_d} = 0,$$
$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\rho_u}{dr}\right) + \frac{\partial V_{\rm eff}}{\partial \rho_u} = 0.$$

b.c.

$$\lim_{r\to\infty} \rho_d(r) = 0, \quad \lim_{r\to\infty} \rho_u(r) = 0,$$
$$\left.\frac{d\rho_d(r)}{dr}\right|_{r=0} = 0, \quad \left.\frac{d\rho_u(r)}{dr}\right|_{r=0} = 0.$$



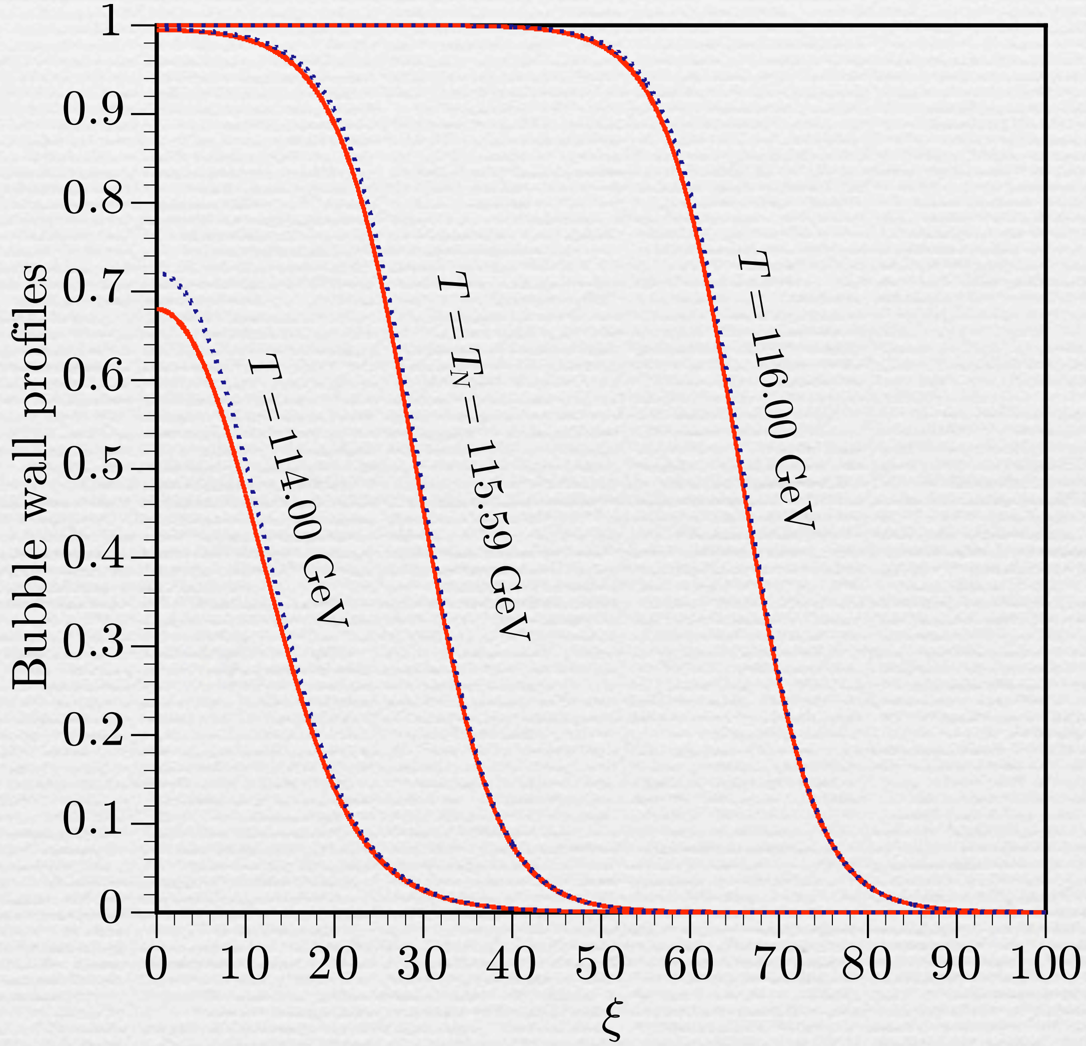
- The EOMs are numerically solved by relaxation methods.

Bubble nucleation

Nucleation rate: $\Gamma_N(T) \simeq T^4 \left(\frac{E_{\rm cb}(T)}{2\pi T}\right)^{3/2} e^{-E_{\rm cb}(T)/T}$

[A.D. Linde, NPB216 ('82) 421]

Nucleation T: $\Gamma_N(T_N)H(T_N)^{-3} = H(T_N)$



$\xi = vr,\ h_1(\xi) = \frac{\rho_d(r)}{v\cos\beta},\ h_2(\xi) = \frac{\rho_u(r)}{v\sin\beta}$

Numerical results:

$$\frac{v_N}{T_N} = \frac{116.73}{115.59} = 1.01$$

10% enhancement! But,

Sphaleron decoupling cond.@T_N:

$\mathcal{E} = 1.77,\ \mathcal{N}_{\rm tr} = 6.65,\ \mathcal{N}_{\rm rot} = 12.27$

$$\frac{v_N}{T_N} > 1.38$$

- The sphaleron process is not decoupled at T_N either.

More examples

$A_b = A_t = -300$ GeV, $m_{\tilde{t}_R} = 10^{-4}$ GeV, $m_{\tilde{b}_R} = 1000$ GeV,
$\mu = 100$ GeV, $M_2 = 500$ GeV,

$m_{\tilde{q}}$ (GeV)	1200	1300	1400	1500
$\tan\beta$	10.11	9.87	9.75	9.57
$m_{H^\pm}$ (GeV)	127.40	127.40	127.50	127.50
v_C/T_C	$\frac{107.096}{116.274} = 0.921$	$\frac{107.512}{116.496} = 0.923$	$\frac{107.769}{116.770} = 0.923$	$\frac{107.915}{117.045} = 0.922$
$\tan\beta_C$	13.803	13.640	13.597	13.455
v_N/T_N	$\frac{116.727}{115.585} = 1.010$	$\frac{117.155}{115.798} = 1.012$	$\frac{117.404}{116.067} = 1.012$	$\frac{117.531}{116.339} = 1.010$
$\tan\beta_N$	13.676	13.503	13.453	13.307
$E_{\rm cb}/T_N$	150.386	150.379	150.370	150.360
$\mathcal{E}$	1.769	1.770	1.770	1.771
$\mathcal{N}_{\rm tr}$	6.652	6.658	6.662	6.667
$\mathcal{N}_{\rm rot}$	12.266	12.253	12.240	12.229
$v_N/T_N >$	1.383	1.382	1.382	1.380

Typically, $v_N/T_N > 1.38$ is needed for sphaleron decopuling.

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$v_N/T_N >$	1.383	1.382	1.382	1.380

Typically, $v_N/T_N > 1.38$ is needed for sphaleron decopuling.

More examples

$A_b = A_t = -300$ GeV, $m_{\tilde{t}_R} = 10^{-4}$ GeV, $m_{\tilde{b}_R} = 1000$ GeV,
$\mu = 100$ GeV, $M_2 = 500$ GeV,

$m_{\tilde{q}}$ (GeV)	1200	1300	1400	1500
$\tan\beta$	10.11	9.87	9.75	9.57
$m_{H^\pm}$ (GeV)	127.40	127.40	127.50	127.50
v_C/T_C	$\frac{107.096}{116.274} = 0.921$	$\frac{107.512}{116.496} = 0.923$	$\frac{107.769}{116.770} = 0.923$	$\frac{107.915}{117.045} = 0.922$
$\tan\beta_C$	13.803	13.640	13.597	13.455
v_N/T_N	$\frac{116.727}{115.585} = 1.010$	$\frac{117.155}{115.798} = 1.012$	$\frac{117.404}{116.067} = 1.012$	$\frac{117.531}{116.339} = 1.010$
$\tan\beta_N$	13.676	13.503	13.453	13.307
$E_{\rm cb}/T_N$	150.386	150.379	150.370	150.360
$\mathcal{E}$	1.769	1.770	1.770	1.771
$\mathcal{N}_{\rm tr}$	6.652	6.658	6.662	6.667
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Typically, $v_N/T_N > 1.38$ is needed for sphaleron decopuling.

Loopholes

Our negative results might be circumvented.

- $T_N \Rightarrow$ onset of the PT. We should know a temperature at which the PT ends. The sphaleron decoupling condition should be imposed at such a temperature.

- Higher order (2-loop) contributions must be taken into account. [J.R. Espinosa, NPB475, ('06) 273]

$\Rightarrow$ The sphaleron decoupling cond. might be relaxed.

- The potential can be extended in such a way that stop also has a nontrivial VEV. (Charge-Color-Breaking vacuum)

  $\Rightarrow$ MSSM BG is viable. [Canena et al, NPB812,('09) 243]
  [N.B.] EW vacuum: metastable, CCB vacuum: global minimum

If the refined sphaleron decoupling cond. is used, is it still viable?

Summary

- We have analyzed the strength of the 1st order EWPT in the light Higgs boson scenario of the MSSM.
- v/T at T_N can be enhanced by about 10% compared to that at T_C.
- The sphaleron decoupling condition at T_N is typically given by $v/T > 1.38$.
- The sphaleron process is not decoupled at both T_C and T_N.

Backup

Decoupling limit

$|A_t| = |A_b| = |\mu|/\tan\beta$, $m_{\tilde{t}_R} = 10^{-4}$ GeV, $m_{\tilde{b}_R} = 1000$ GeV, $|\mu| = 100$ GeV, $M_1 = 100$ GeV, $M_2 = 500$ GeV.

$m_{\tilde{q}}$ (GeV)	1700	1800	1900	2000
$\tan\beta$	42.62	15.10	10.97	9.35
$m_{H^\pm}$ (GeV)	1000.00	1000.00	1000.00	1000.00
v_C/T_C	$\frac{111.461}{116.993} = 0.953$	$\frac{111.460}{117.007} = 0.953$	$\frac{111.483}{116.994} = 0.953$	$\frac{111.440}{117.060} = 0.952$
$\tan\beta_C$	42.966	15.171	11.022	9.394
v_N/T_N	$\frac{121.454}{116.221} = 1.045$	$\frac{121.452}{116.236} = 1.045$	$\frac{121.478}{116.222} = 1.045$	$\frac{121.424}{116.288} = 1.044$
$\tan\beta_N$	42.955	15.168	11.019	9.392
$E_{\rm cb}(T_N)/T_N$	150.366	150.370	150.364	150.360
$\mathcal{E}$	1.773	1.773	1.773	1.773
$\mathcal{N}_{\rm tr}$	6.677	6.677	6.678	6.678
$\mathcal{N}_{\rm rot}$	12.211	12.210	12.210	12.209
$v_N/T_N >$	1.379	1.379	1.379	1.379

The sphaleron process is not decoupled in this case either.