Flavor Physics
in the Littlest Higgs Model with T-Parity

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The 8th Particle Physics Phenomenology Workshop @NCKU, May 20,2009

Phys.Lett.B670:378-382,2009.
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Introduction

- Motivations and plan of this talk.

The Littlest Higgs Model with T-Parity

- Details of the model which we discuss.

Impact to Flavor Physics

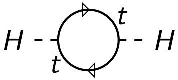
- Numerical calculation of $K \to \pi\nu\bar{\nu}$ and lepton flavor violation.

Conclusion

Hierarchy Problems

- **Hierarchy Problem**

  - In the SM, physical Higgs mass can be extremely smaller than radiative corrections.



$$\rightarrow \delta m^2 \sim (0.3\, \Lambda_{cut\ off})^2$$

  - Unnaturalness increase at $\Lambda_{cut\ off} \gtrsim 1$ TeV.

- **Little Hierarchy Problem**

  - Electroweak precision measurement, in LEP, implies the scale of new physics $\gtrsim$ 5 TeV.

$$\frac{\delta m^2}{m_H^2} \gtrsim \left(0.3\, \frac{5\text{TeV}}{100\text{GeV}}\right)^2 \sim 200$$

'99 R.Barbieri, A.Strumia

The Littlest Higgs Model with T-Parity

- **The Littlest Higgs Model**

  - The Higgs boson is realized as a pseudo Nambu-Goldstone boson.

  - Quadratic divergences generated by the top quark and gauge bosons can be compensated with corresponding heavy new particles at 1-loop.

  '01 N.Arkani-Hamed, A.G.Cohen, H.Georgi

  '02 N.Arkani-Hamed, A.G.Cohen, E.Katz, A.E.Nelson

- **T-Parity**

  - To avoid extra contributions to 4 fermi interaction against electroweak precision constraints, symmetry breaking scale become large i.e. little hierarchy problem revives.

  - Z_2 symmetry, T-parity, suppress the contributions.
  - New particles are needed to impose T-parity.

  '03 C.H.Chen, I.Low

Impact to Flavor Physics

- **New Fermion Mixing Matrices**
  - New contributions to flavor signals are induced.

'05 J.Hubisz, S.J.Lee, G.Paz

'06 M.Blanke, A.J.Buras, S.Poschenrieder, S.Recksiegel, C.Tarantino, S.Uhlig, A.Weiler

- **Previous Works**
  - Low energy ultra-violet sensitivity is reported in loop induced flavor changing processes.

'06∼'07 M.Blanke, A.J.Buras, C.Tarantino et.al.

- **Our Results**
  - Additional terms missing in previous works are found.
  - Low energy ultra-violet sensitivity is cancelled.
  - Numerical calculations of $K \to \pi\nu\bar{\nu}$ and lepton flavor violation.

'08 T.Goto, Y.Okada, Y.Y.

Global Symmetry

- A non-linear sigma model, $SU(5) \xrightarrow{f} SO(5)$.

  - Vaccum expectation value of Σ, Nambu-Goldstone bosons, break the global symmetry.
  - Spontaneous symmetry breaking generate masses of new particles, $\mathcal{O}(f)$.
  - A effective theory below the cut off scale, $4\pi f$.

- There are 14 Nambu-Goldstone bosons.

  - The Higgs boson is realized as a pseudo Nambu-Goldstone boson.

$$\Sigma = \xi \Sigma_0 \xi^T, \quad \Sigma_0 = \begin{pmatrix} & & \mathbf{1} \\ & 1 & \\ \mathbf{1} & & \end{pmatrix}$$

$$\xi = \exp\left[\frac{i}{f} \begin{pmatrix} \omega & H & \Phi \\ H^\dagger & \eta & H^T \\ \Phi^\dagger & H^* & \omega \end{pmatrix} \right]$$

Gauge Symmetry

- Subgroups of global symmetry, $SU(5)$, are gauged.

$$\begin{array}{ccc} SU(5) & \xrightarrow{f} & SO(5) \\ \cup & & \cup \\ [SU(2)\times U(1)]_1 \times [SU(2)\times U(1)]_2 & \xrightarrow{f} & SU(2)_L \times U(1)_Y \end{array}$$

- $[SU(2)\times U(1)]_1 \overset{T-parity}{\longleftrightarrow} [SU(2)\times U(1)]_2$

  $\Rightarrow W^{\pm}$, Z, A (T-even), $W_H^{\pm}$, Z_H, A_H (T-odd).

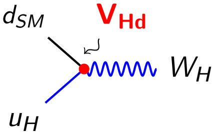
- Pseudo Nambu-Goldstone bosons.

  - H : The SM Higgs doublet (T-even).
  - ω,η : Eaten by heavy gauge bosons (T-odd).
  - ϕ : Physical heavy scalar (T-odd).

Fermions

- Left-handed fermions consist of **5** and $\overline{\mathbf{5}}$ of $SU(5)$.
  $\Psi_1(\mathbf{5}) \overset{T-parity}{\longleftrightarrow} -\Sigma_0\Psi_2(\overline{\mathbf{5}})$
- Right-handed heavy fermions are **5** of $SO(5)$, and non-linear representations of $SU(5)$.
  - $\Psi'_R = \xi\Psi_R$ transforms as **5** of $SU(5)$.
- Yukawa coupling generate $\mathcal{O}(f)$ masses for T-odd fermions.
  $\mathcal{L}_{HY} \supset -\kappa^{ij} f(\overline{\Psi}{}_2^i \Psi'^j_R - \overline{\Psi}{}_1^i \tilde{\Psi}'^j_R) + H.c.$
  ($\tilde{\Psi}'_R$ is T-parity conjugate of Ψ'_R)
- New mixing matrices are induced between the SM fermions and heavy new fermions since mass bases of heavy fermions are independent of that of the SM fermions.

$$\frac{g}{\sqrt{2}}(\mathbf{V_{Hd}})_{ij}\ u^i_H \gamma^\mu W^+_{H\,\mu} d^j_{SM} \quad \sim$$



Previous Results

- Flavor signals were calculated with low energy expansion of the full Lagurangean.

  - 4 dimension relevant operators.

  - Including corrections until $\mathcal{O}(v^2/f^2)$.

  - Leading contributions of new particles are $\mathcal{O}(v^2/f^2)$.

- In the amplitudes of loop induced flavor changing processes, low energy ultra-violet sensitivity were reported.

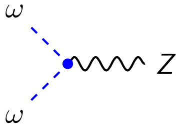
  - Loss of low energy predictability.

$$Br(\text{flavor changing})_{T\text{-}odd} \sim \frac{v^2}{f^2}\left(\ln\frac{\Lambda^2}{M_{WH}^2} + \text{finite}\right)$$

Corrections of Non-Linear Representation

- There are $\mathcal{O}(v^2/f^2)$ contributions in the expansion of non-linear Lagrangean.

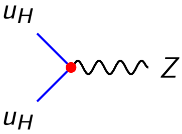
  $\mathcal{L} \supset \frac{f^2}{8}\mathrm{tr}[(D^\mu\Sigma)^\dagger D_\mu\Sigma]$

  $\longrightarrow$ 

  $\propto \left(\cos^2\theta_W - \frac{\mathbf{v^2}}{\mathbf{8f^2}}\right)$

- Missing terms are found in the expansion of interactions between right-handed heavy fermions, non-linear representation, and Z boson.

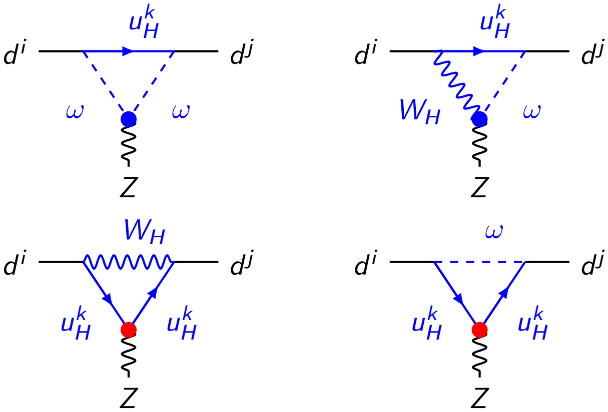
  $\mathcal{L} \supset \bar{\Psi}_R\xi^\dagger\, iD\!\!\!/\, \xi\Psi_R, \quad \Psi_R : \mathbf{5}$ of $SO(5)$

  $\longrightarrow$ 

  $\propto \left[\left(\frac{1}{2} - \frac{2}{3}s_W^2\right)\gamma^\mu - \frac{\mathbf{v^2}}{\mathbf{8f^2}}\boldsymbol{\gamma^\mu}\mathbf{P_R}\right]$

Cancelation of Ultara-Violet Sensitivity

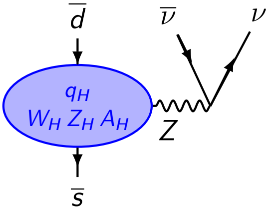
- Relevant diagrams for leading $\mathcal{O}(v^2/f^2)$ contributions.

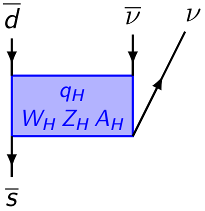


- Reported low energy ultra-violet sensitivity, contribution of blue vertices, is canceled by new contributions, red vertices.

$K \rightarrow \pi \nu \overline{\nu}$

- This processes receive small QCD correction.
  - Theoriticaly clean and sensitive to new physics.
- $K^+ \rightarrow \pi^+ \nu \overline{\nu}$
  - A few events have been observed.
- $K_L \rightarrow \pi^0 \nu \overline{\nu}$
  - Any events have not been observed.
  - In J-PARC, new experiment is planned.
- New contributions in the littlest Higgs model with T-parity.





Numerical Calculation

- There are **20** new parameters in the littlest Higgs model with T-parity.

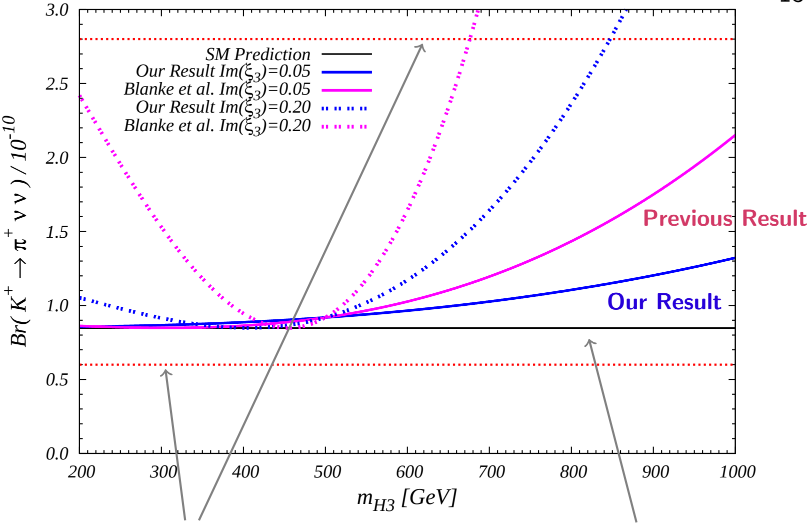
f, m_T, 6 fermion masses, 6 mixing angles, 6 phases

$$f = 1 \text{ TeV}\ ,\ m_T = 1.34 \text{ TeV},$$
$$m_{Hq}^{1,2} = m_{H\ell}^{i} = 500 \text{ GeV},$$
$$\text{Re}[\xi_3] = 0, \qquad (\boldsymbol{\xi}_3 = (\mathbf{V}_{\mathbf{Hd}}^{*})_{\mathbf{3s}}(\mathbf{V}_{\mathbf{Hd}})_{\mathbf{3d}}).$$

- Dengerous contributions to K^0-$\overline{K^0}$ are suppressed.

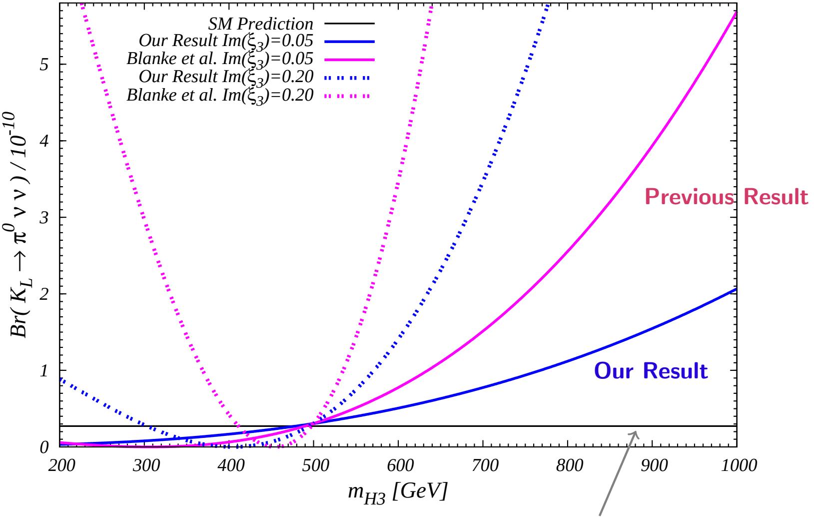
$\mathbf{m_{Hq}^3}$ and $\mathbf{Im}[\boldsymbol{\xi}_\mathbf{3}]$ are free parameters.

Numerical Calculation of $K^+ \to \pi^+ \nu \overline{\nu}$



$Br(K^+ \to \pi^+ \nu \bar{\nu})_{\text{ex}} = (1.47^{+1.30}_{-0.89}) \times 10^{-10}$ $Br(K^+ \to \pi^+ \nu \bar{\nu})_{\text{SM}} = 8.5 \times 10^{-11}$

Numerical Calculation of $K_L \rightarrow \pi^0 \nu \overline{\nu}$

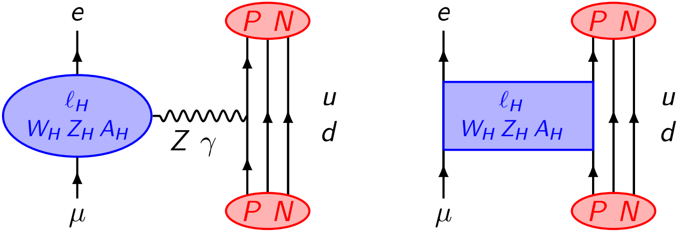


$Br(K_L \rightarrow \pi^0\ \nu\ \bar{\nu})_{\text{ex}} < 2.1 \times 10^{-7}$

$Br(K_L \rightarrow \pi^0\ \nu\ \bar{\nu})_{\text{SM}} = 2.7 \times 10^{-11}$

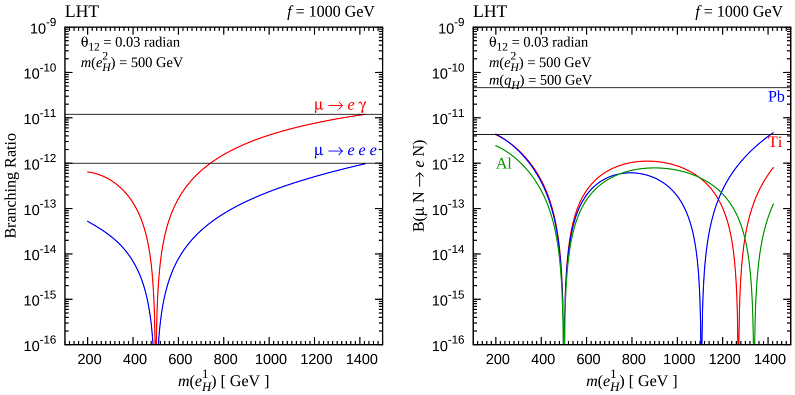
Lepton Flavor Violation

- In the SM, branching ratios become extreamely small.
  - Observation means new physics.
- Experiments.
  - μ - e γ experiment in MEG.
  - $\mu \rightarrow$ 3e is bounded by SINDRUM.
  - μ - e conversion is planned in FNAL and J-PARC.
- New contributions.



- Lepton sector is similar to quark sector in this model.
  - Using $m^1_{H\ell}$ and θ_{12}, in $V_{H\ell}$, as free parameter.

Numerical Calculation of Lepton Flavor Violation



- $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ have been studied.

'07 M.Blanke, A.Buras, B.Duling, A.Poschenrieder and C.Tarantino

'09 F.del Aguila, J.I.Illana and M.D.Jenkins

- Matrix elements for μ - e conversion are taken from known formalism.

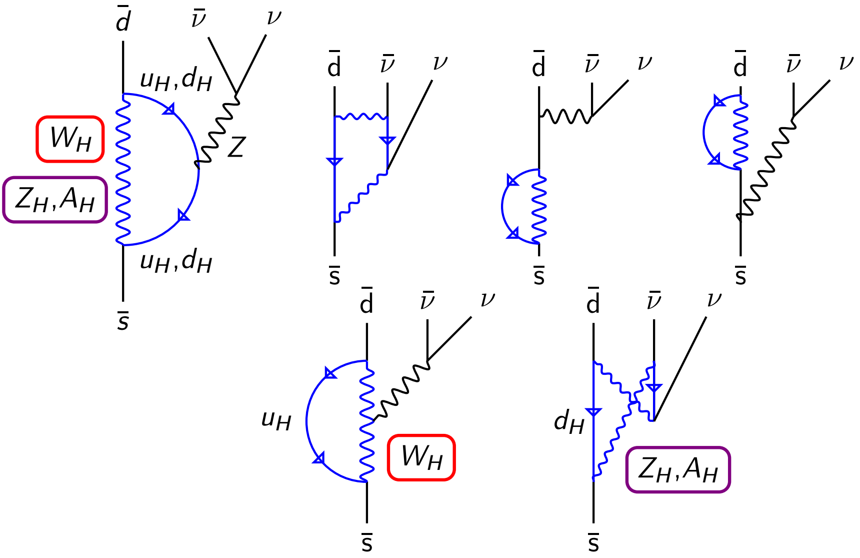
'02 R.Kitano, M.Koike and Y.Okada

Conclusion

- The littlest Higgs model with T-parity has charastristic flavor signals.
- Amplitudes of loop induced flavor changing processes, $d^i \longrightarrow d^j$ and $e^i \longrightarrow e^j$, become ultra-violet finite at $\mathcal{O}(\frac{v^2}{f^2})$.
- Branching ratios of $K \longrightarrow \pi\nu\overline{\nu}$ can be different from the SM predictions.
- Lepton flavor violation can be large enough to be measurable.
- The processes may be measured in near future experiment.

Back up Slides

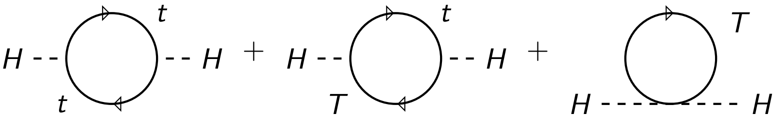
New Contributions to $K \to \pi\nu\bar{\nu}$



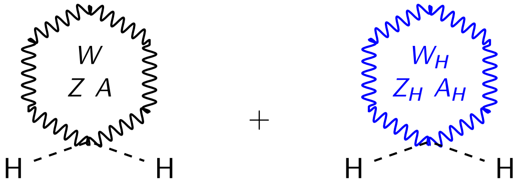
Collective Symmetry Breaking

- Quadratic divergences are compensated with **new particles** at 1-loop.

top quark ↔ top partner



gauge bosons ↔ heavy gauge bosons



Nambu-Goldstone Bosons

$$\xi = \exp\left[i\Pi/f\right]$$

$$\Pi = \begin{pmatrix} \frac{\omega^0}{2} - \frac{\eta}{2\sqrt{5}} & -\frac{\omega^+}{\sqrt{2}} & -i\frac{\pi^+}{\sqrt{2}} & -i\phi^{++} & -i\frac{\phi^+}{\sqrt{2}} \\ -\frac{\omega^+}{\sqrt{2}} & \frac{\omega^0}{2} - \frac{\eta}{2\sqrt{5}} & \frac{v+h+i\pi^0}{2} & -i\frac{\phi^+}{\sqrt{2}} & \frac{-i\phi^0+\phi^{\mathrm{P}}}{\sqrt{2}} \\ i\frac{\pi^-}{\sqrt{2}} & \frac{v+h-i\pi^0}{2} & \frac{2\eta}{\sqrt{5}} & -i\frac{\pi^+}{\sqrt{2}} & \frac{v+h+i\pi^0}{2} \\ i\phi^{--} & i\frac{\phi^-}{\sqrt{2}} & i\frac{\pi^-}{\sqrt{2}} & -\frac{\omega^0}{2} - \frac{\eta}{2\sqrt{5}} & -\frac{\omega^-}{\sqrt{2}} \\ i\frac{\phi^-}{\sqrt{2}} & \frac{i\phi^0+\phi^{\mathrm{P}}}{\sqrt{2}} & \frac{v+h-i\pi^0}{2} & -\frac{\omega^+}{\sqrt{2}} & \frac{\omega^0}{2} - \frac{\eta}{2\sqrt{5}} \end{pmatrix}$$

U(1) Charge Assignment to Left-handed Fermions

	$SU(2)_1$	$SU(2)_2$	Y_1	Y_2	Y_1'	Y_2'	Y_1''	Y_2''
$q_1 = \begin{pmatrix} u_1 \\ d_1 \end{pmatrix}$	$\mathbf{2}$	$\mathbf{1}$	$\frac{1}{30}$	$\frac{4}{30}$	$-\frac{3}{10}$	$-\frac{2}{10}$	$\frac{1}{3}$	$\frac{1}{3}$
$q_2 = \begin{pmatrix} u_2 \\ d_2 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$\frac{4}{30}$	$\frac{1}{30}$	$-\frac{2}{10}$	$-\frac{3}{10}$	$\frac{1}{3}$	$\frac{1}{3}$
t_1'	$\mathbf{1}$	$\mathbf{1}$	$\frac{8}{15}$	$\frac{2}{15}$	$\frac{2}{10}$	$-\frac{2}{10}$	$\frac{1}{3}$	$\frac{1}{3}$
t_2'	$\mathbf{1}$	$\mathbf{1}$	$\frac{2}{15}$	$\frac{8}{15}$	$-\frac{2}{10}$	$\frac{2}{10}$	$\frac{1}{3}$	$\frac{1}{3}$
$\ell_1 = \begin{pmatrix} \nu_1 \\ e_1 \end{pmatrix}$	$\mathbf{2}$	$\mathbf{1}$	$-\frac{2}{10}$	$-\frac{3}{10}$	$-\frac{3}{10}$	$-\frac{2}{10}$	0	0
$\ell_2 = \begin{pmatrix} \nu_2 \\ e_2 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{3}{10}$	$-\frac{2}{10}$	$-\frac{2}{10}$	$-\frac{3}{10}$	0	0

U(1) Charge Assignment to Right Handed Fermions

	$SU(2)_1$	$SU(2)_2$	Y_1	Y_2	Y_1'	Y_2'	Y_1''	Y_2''
u_R	$\mathbf{1}$	$\mathbf{1}$	$\frac{1}{3}$	$\frac{1}{3}$	0	0	$\frac{1}{3}$	$\frac{1}{3}$
d_R	$\mathbf{1}$	$\mathbf{1}$	$-\frac{1}{6}$	$-\frac{1}{6}$	0	0	$-\frac{1}{6}$	$-\frac{1}{6}$
$q_{HR} = \begin{pmatrix} u_{HR} \\ d_{HR} \end{pmatrix}$	–	–	–	–	–	–	$\frac{1}{3}$	$\frac{1}{3}$
t'_{1R}	$\mathbf{1}$	$\mathbf{1}$	$\frac{8}{15}$	$\frac{2}{15}$	$\frac{2}{10}$	$-\frac{2}{10}$	$\frac{1}{3}$	$\frac{1}{3}$
t'_{2R}	$\mathbf{1}$	$\mathbf{1}$	$\frac{2}{15}$	$\frac{8}{15}$	$-\frac{2}{10}$	$\frac{2}{10}$	$\frac{1}{3}$	$\frac{1}{3}$
ν_R	$\mathbf{1}$	$\mathbf{1}$	0	0	0	0	0	0
e_R	$\mathbf{1}$	$\mathbf{1}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$
$\ell_{HR} = \begin{pmatrix} \nu_{HR} \\ e_{HR} \end{pmatrix}$	–	–	–	–	–	–	0	0