$B \to K\eta, K\eta'$ Decays in QCD Factorization

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1 Introduction

• In charmless two-body decays, the $B \to K\eta'$ decay is the one with the largest branching ratio, bigger than that of $B \to K\pi$ decay by a factor of ≈ 3.

• The $B \to K\eta, K\eta'$ decays have been analysed in many recent papers, for example, Beneke and Neubert, NP B 651, 225 (2003); Dutta, Kim, Oh and Zhu, EPJC 37 , 273 (2004), Williamson and Zupan, PRD 74 , 014003(2006); Charng, Kurimoto and Li, PRD 74 074024 (2006); Gerard and Kou, PRL 97 261804 (2006) and more recently Hsu, Charng and Li, PRD 78 014020 (2008) and Xiao, Liu and Guo, PRD 78 114001 (2008).

• In QCDF the $B \to K\pi$ branching ratio could be understood with a moderate contribution from the power-suppressed annihilation terms.

• Without fine tuning, the $B \to K\eta'$ branching ratio is predicted to be larger than that of $B \to K\pi$, but still underestimated by $20 - 30\%$.

- Main theoretical uncertainties come from the $B \to \eta'$ form factor and the pseudo-scalar density matrix elements for η'.
- This work : To show that nonet symmetry for the pseudo-scalar mass formula implies nonet symmetry for the pseudo-scalar density matrix elements and to use this result in this analysis of $B \to K\eta, K\eta'$ decays.
- Historically, there is an approximate expression for the octet pseudo-scalar density matrix elements by Gell-Mann, Oakes and Renner, PRD 175, 2195 (1968).
- There is no known explicit expression for the singlet pseudo-scalar density using the nonet symmetry scheme.

2 QCD factorization for charmless $B \to PP$ decays

Figure 1: factorization diagram for charmless B decay

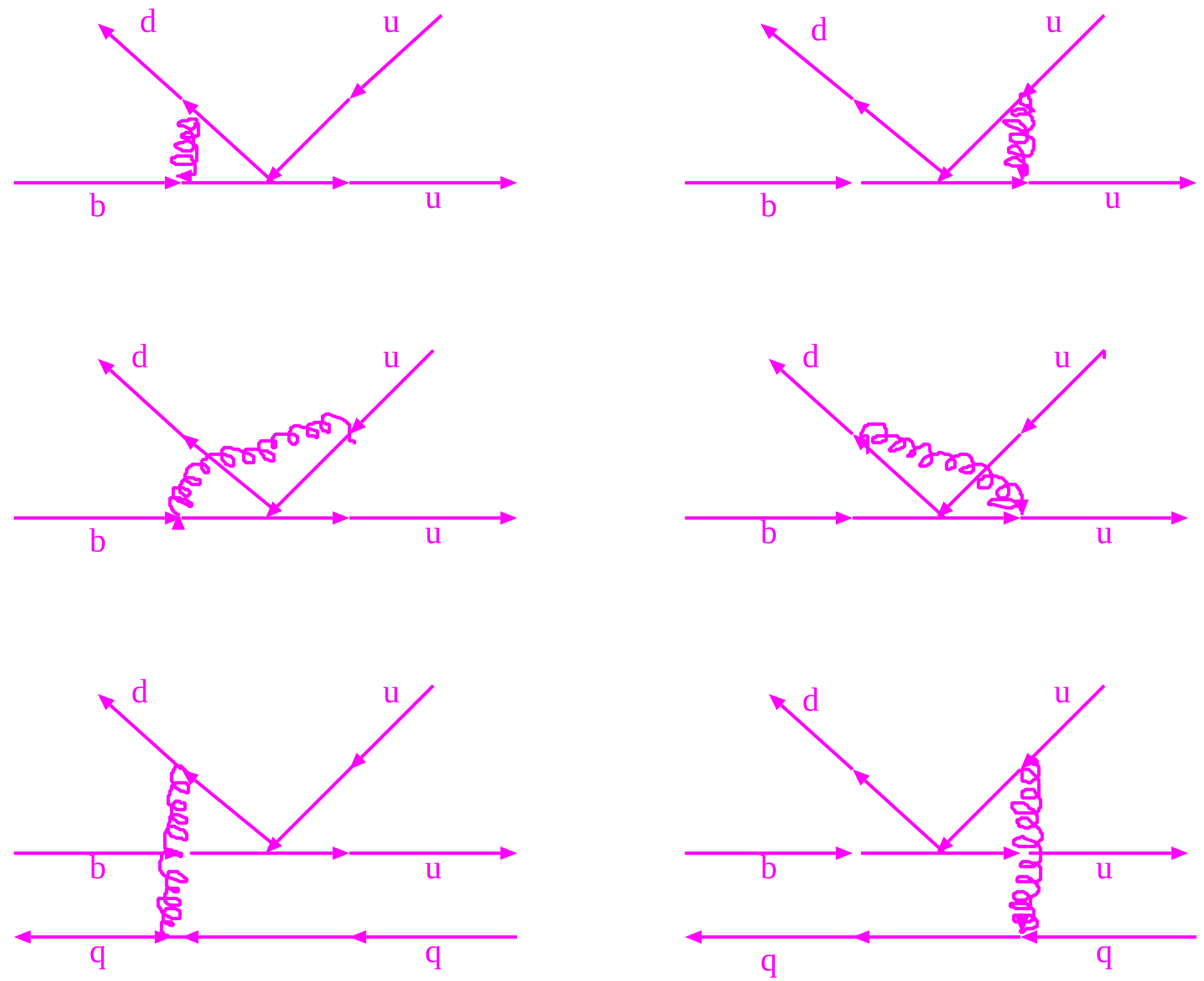

Figure 2: Vertex and spectator corrections to charmless B decays

Figure 3: Annihilation diagram for $B \to M_1 M_2$

• With the operator product expansion and renormalization group equation, the effective Lagrangian can be obtained, in which short-distance effects involving large virtual momenta of the loop corrections from the scale M_W down to $\mu = \mathcal{O}(m_b)$ are cleanly integrated into the Wilson coefficients.

• $B \to M_1 M_2$ decay amplitude

$$\mathcal{A}(B \to M_1 M_2) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{ps}^* \times$$

$$\left(- \sum_{i=1}^{10} a_i^p \langle M_1 M_2 | O_i | B \rangle_H + \sum_i^{10} f_B f_{M_1} f_{M_2} b_i \right) , \qquad (1)$$

• The QCD coefficients a_i^p contain the vertex corrections, penguin corrections, and hard spectator scattering contributions, the hadronic matrix elements $\langle M_1 M_2 | O_i | B \rangle_H$ of the tree and penguin operators O_i are given by factorization model, b_i are annihilation contributions. The values for a_i^p,$p = u, c$, evaluated at the renormalization scale $\mu = m_b$, with $m_b = 4.2\,\mathrm{GeV}$

• The effective operators :

$$O_1 = (\bar{s}u)_L (\bar{u}b)_L \quad , \; O_4 = \sum_q (\bar{s}q)_L (\bar{q}b)_L$$

$$O_6 = -2 \sum_q (\bar{s}_L q_R)(\bar{q}_R b_L) \qquad (2)$$

• $O_{1,2}$: Tree-level, $O_{3,\ldots,6}$: QCD penguin, $O_{7,\ldots,10}$: Electroweak penguin

• Hadronic matrix elements : $\langle M_1 M_2 | O_i(\mu) | B \rangle$ contains the physics effects from the scale $\mu = \mathcal{O}(m_b)$ down to $\Lambda_{\rm QCD}$.

• In heavy quark limit, QCD Factorisation : $\langle M_1 M_2 | O_i(\mu) | B \rangle$ can be factorized into hard radiative corrections and simpler nonperturbative structures which can be parametrized by the form factors and meson light-cone distribution amplitudes (LCDAs).

• Power corrections in $1/m_b$ come from Penguin matrix element, chirally enhanced corrections and annihilation contribution.

• Penguin matrix element like that of O_6 is of the order $O(1/m_b)$ compared to the $(V-A)\times(V-A)$ O_1 and O_4 matrix element in the $B \to K\pi$ amplitude, since the matrix element $< K|\bar{s}_L d_R|0 >$ is proportional to $m_K^2/m_s \approx 2.5\,\mathrm{GeV}$ while $< K|\bar{s}_L d_L|0 >$ is proportional to K momentum which is $O(m_b)$, thus numerically, the matrix element of O_6 which has a factor

$$r_\chi^K = \frac{2m_K^2}{m_b(m_s+m_d)} \approx O(1) \qquad (3)$$

is comparable to that of O_4.

• Chirally enhanced corrections from two-body twist-3 LCDAs of the final state mesons and annihilation contributions parametrized by the two quantities $X_{\mathrm{A,H}}$ which could have a strong phase (M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, Nucl. Phys. B **606**, 245 (2001).)

3 Nonet symmetry in the $\eta - \eta'$ system

• Since QCD interactions through the exchange of gluons are flavor-independent, one expects the wave function for the pseudo-scalar meson nonet also flavor-independent in the limit of vanishsing current quark mass.

• η and η' can be described as two linear combinations of the $q\bar{q}$ state, the SU(3) singlet η_0 and the SU(3) octet η_8 which mix with each other through a small $SU(3)$ symmetry breaking mixing parameter.

• because of the $U(1)$ QCD-anomaly, the η_0 mass is much larger compared to the η_8 mass, the $\eta - \eta'$ mixing angle is $O(m_s/\Lambda_{\mathrm{QCD}})$.

• The quark mass term is the leading term in the large N_c expansion while higher order terms in the chiral Lagrangian (Gasser and Leutwyler) is $O(1/N_c)$ and is thus suppressed in the large N_c limit.

• This justifies the nonet symmetry scheme for the pseudo-scalar meson mass term.

• Nonet symmetry for the off-diagonal mass term $< \eta_0|H_{\rm SB}|\eta_8 >$ gives an $\eta - \eta'$ mixing angle $\theta = -18^\circ$ in good agreement with a value $\theta \approx -(22 \pm 3)^\circ$ (Donoghue) or $\theta \approx -(18.4 \pm 2)^\circ$ (Pham) and a similar value $\theta \approx -(17 - 20)^\circ$ (Ball) obtained from the two-photon decay width of η and η'.

• However, from the Gell-Mann-Okubo(GMO) mass formula, we would have

$$m_\eta^2 = m_8^2 - \tan\theta^2\,(m_{\eta'}^2 - m_8^2) \tag{4}$$

which gives, for $\theta = -18^\circ$, $m_\eta = 483\,\mathrm{MeV}$, about $60\,\mathrm{MeV}$ below experiment.

• The $\eta - \eta'$ mixing which contributes to L_7 has driven the m_η below the GMO value by $63\,\mathrm{MeV}$.

• Higher order terms in the chiral Lagrangian and chiral logarithms(Gasser and Leutwyler; Gerard and Kou(2005)), shift m_η upward by a similar amount with the result that the η mass is very close to the GMO value, and a large $\eta - \eta'$ mixing angle consistent with nonet symmetry rather than the small value of -10° obtained with the GMO formula for m_8 .

• Thus nonet symmetry seems to be a good approximation for the 0^- nonet mass term.

• The matrix elements of the pseudo-scalar density local operator *e.g.* $\bar{s}\, i\gamma_5\, s$ might also satisfy nonet symmetry.

• This is in fact the case as shown in this work that nonet symmetry for the mass term implies nonet symmetry for the pseudo-scalar density local operator.

• The penguin matrix elements in charmless B decays with η' in the final state could be then computed using nonet symmetry.

• The matrix element of the axial vector current $\bar{u}\,\gamma_\mu\gamma_5 u$ and $\bar{s}\,\gamma_\mu\gamma_5 s$ between the vacuum and η_0 and η_8 :

$$< 0|\bar{u}\,\gamma_\mu\gamma_5 u|\eta_0 >= i\, f_u\, p_\mu/\sqrt{3}, \quad < 0|\bar{u}\,\gamma_\mu\gamma_5 u|\eta_8 >= i\, f_u\, p_\mu/\sqrt{6}. \qquad (5)$$

$$< 0|\bar{s}\,\gamma_\mu\gamma_5 s|\eta_0 >= i\, f_s\, p_\mu/\sqrt{3}, \quad < 0|\bar{s}\,\gamma_\mu\gamma_5 s|\eta_8 >= -2\, i\, f_s\, p_\mu/\sqrt{6}. \qquad (6)$$

• The octet $A_{8\,\mu}$ and singlet $A_{0\,\mu}$ axial vector current matrix elements

$$< 0|A_{\mu 8}|\eta_8 >= \frac{(f_u + f_d + 4\,f_s)}{6}\,p_\mu, \quad < 0|A_{\mu 0}|\eta_0 >= \frac{(f_u + f_d + f_s)}{3}\,p_\mu. \tag{7}$$

• Assuming each s- quark contributes to the decay constant a symmetry breaking term ϵ, to first order in ϵ(Pham(1984)) (Rewriting $f_{q\bar{q}} = f_q$):

$$\begin{aligned} f_\pi &= f_{u\bar{d}} \approx f_u, \quad f_K = f_{u\bar{s}} = (1+\epsilon)\,f_{u\bar{d}}, \\ f_s &= (1 + 2\,\epsilon)\,f_u \approx (1+\epsilon)\,f_K. \end{aligned} \tag{8}$$

• Pseudo-scalar density matrix elements for the pseudo-scalar meson octet:

$$\begin{aligned} f_\pi B_0(m_u + m_d) &= (m_u + m_d)\langle 0|\bar{u}\,i\gamma_5 d|u\bar{d}\rangle, \\ f_K B_0(m_u + m_s) &= (m_u + m_s)\langle 0|\bar{u}\,i\gamma_5 s|u\bar{s}\rangle. \end{aligned} \tag{9}$$

and for π^0 :

$$f_u B_0(m_u + m_d) = (m_u + m_d)\langle 0|\bar{u}\,i\gamma_5 u|u\bar{u}\rangle. \tag{10}$$

• Consider now the $I=0$ $A_{\mathrm{n}\,\mu}$ and $A_{\mathrm{s}\,\mu}$ axial vector current:

$$A_{\mathrm{n}\,\mu} = (\bar{u}\,\gamma_\mu\gamma_5 u + \bar{d}\,\gamma_\mu\gamma_5 d), \quad A_{\mathrm{s}\,\mu} = \bar{s}\,\gamma_\mu\gamma_5 s. \tag{11}$$

• The divergence:

$$\partial A_{\mathrm{n}} = 2(m_u \bar{u} i\gamma_5 u + m_d \bar{d} i\gamma_5 d) + 2\frac{\alpha_s}{4\pi} G\,\tilde{G}. \tag{12}$$

$$\partial A_{\mathrm{s}} = 2m_s \bar{s} i\gamma_5 s + \frac{\alpha_s}{4\pi} G\,\tilde{G}. \tag{13}$$

• The matrix elements of ∂A_{n} and ∂A_{s} between the vacuum and $\eta_{0,8}$ are given by:

$$\begin{aligned}\langle 0|\partial A_{\mathrm{n}}|\eta_0\rangle &= 2m_u\langle 0|\bar{u}\,i\gamma_5 u|\eta_0\rangle + 2m_d\langle 0|\bar{d}\,i\gamma_5 d|\eta_0\rangle, \\ &+ \quad 2\langle 0|\frac{\alpha_s}{4\pi} G\,\tilde{G}|\eta_0\rangle,\end{aligned} \tag{14}$$

$$\langle 0|\partial A_{\mathrm{s}}|\eta_0\rangle = 2m_s\langle 0|\bar{s}\,i\gamma_5 s|\eta_0\rangle + \langle 0|\frac{\alpha_s}{4\pi} G\,\tilde{G}|\eta_0\rangle. \tag{15}$$

$$\begin{aligned}\langle 0|\partial A_{\mathrm{n}}|\eta_8\rangle &= 2m_u\langle 0|\bar{u}\,i\gamma_5 u|\eta_8\rangle + 2m_d\langle 0|\bar{d}\,i\gamma_5 d|\eta_8\rangle, \\ &+ \quad 2\langle 0|\frac{\alpha_s}{4\pi} G\,\tilde{G}|\eta_8\rangle,\end{aligned} \tag{16}$$

$$\langle 0|\partial A_s|\eta_8\rangle \;=\; 2m_s\langle 0|\bar{s}\, i\gamma_5 s|\eta_8\rangle + \langle 0|\frac{\alpha_s}{4\pi} G\,\tilde{G}|\eta_8\rangle. \qquad (17)$$

• Consider next Eq.(14-15) and Eq.(16-17) with the pole terms included using the nonet symmetry expressions ($\hat{m} = (m_u + m_d)/2$).

$$\begin{aligned} m_8^2 &= B_0\frac{2}{3}\,(2m_s + \hat{m}), \\ m_0^2 &= \bar{m}_0^2 + B_0\frac{2}{3}(m_s + 2\hat{m}), \\ m_{08}^2 &= B_0\frac{2}{3}\sqrt{2}(-m_s + \hat{m}). \end{aligned} \qquad (18)$$

• The pseudo-scalar density matrix elements can now be extracted from the following expressions

$$\begin{aligned} & f_u\frac{1}{\sqrt{3}}(\bar{m}_0^2 + B_0\frac{2}{3}(m_s + 2\hat{m})) = f_u\frac{1}{\sqrt{3}}m_0^2 - \\ & f_u\frac{1}{\sqrt{6}}(B_0\frac{2\sqrt{2}}{3}(\hat{m} - m_s)) + 2\frac{1}{\sqrt{3}}\hat{m}\langle 0|\bar{u}\, i\gamma_5 u|u\bar{u}\rangle, \qquad (19) \\ & f_s\frac{1}{\sqrt{3}}(\bar{m}_0^2 + B_0\frac{2}{3}(m_s + 2\hat{m})) = f_s\frac{1}{\sqrt{3}}m_0^2 - \end{aligned}$$

$$f_s \frac{2}{\sqrt{6}} B_0 \frac{2\sqrt{2}}{3} (\hat{m} - m_s) + 2 \frac{1}{\sqrt{3}} m_s \langle 0 | \bar{s}\, i\gamma_5\, s | s\bar{s} \rangle . \qquad (20)$$

- Thus for η_0:

$$\langle 0 | \bar{u}\, i\gamma_5 u | u\bar{u} \rangle = B_0 f_u, \quad \langle 0 | \bar{s}\, i\gamma_5\, s | s\bar{s} \rangle = B_0 f_s . \qquad (21)$$

- Similarly, for η_8 :

$$f_u \frac{1}{\sqrt{6}} (B_0 \frac{2}{3} (2m_s + \hat{m})) = -f_u \frac{1}{\sqrt{3}} B_0 \frac{2\sqrt{2}}{3} (\hat{m} - m_s) + 2 \frac{1}{\sqrt{6}} \hat{m} \langle 0 | \bar{u}\, i\gamma_5 u | u\bar{u} \rangle , \qquad (22)$$

$$-f_s \frac{2}{\sqrt{6}} (B_0 \frac{2}{3} (2m_s + \hat{m})) = -f_s \frac{1}{\sqrt{3}} B_0 \frac{2\sqrt{2}}{3} (\hat{m} - m_s) - 2 \frac{2}{\sqrt{6}} m_s \langle 0 | \bar{s}\, i\gamma_5\, s | s\bar{s} \rangle . \qquad (23)$$

- In the limit $m_u = m_d = 0$, the l.h.s and r.h.s of Eq. (22) become $f_u\, m_8^2/\sqrt{6}$ in agreement with the divergence equation Eq. (12).

- from the above equations, we have the same expression for the pseudo-scalar density matrix element, but in η_8 .

• Like $\langle 0|\bar{u}\, i\gamma_5 d|\pi^+\rangle$, $\langle 0|\bar{u}\, i\gamma_5 u|\pi^0\rangle$ and $\langle 0|\bar{u}\, i\gamma_5 s|K^+\rangle$, they are given by the parameter B_0 and the decay constant involved.

• Experimentally, $m_{08}^2 = -(0.81 \pm 0.05)\, m_K^2$ to be compared with the nonet symmetry value of $m_{08}^2 \simeq -0.90\, m_K^2$(Donoghue).

• Expect nonet symmetry for the pseudo-scalar density matrix elements in $\eta - \eta'$ valid to this accuracy.

• Since the octet m_8^2 mass gets about 15% increase from higher order terms L_4, L_5, L_6, L_8 and chiral logarithms, Eqs.(22-23) show that $\langle 0|\bar{s}\, i\gamma_5 s|s\bar{s}\rangle$ in η will be increased by a similar amount

• Assuming a similar 15% increase from the nonet value for m_0^2, $\langle 0|\bar{s}\, i\gamma_5 s|s\bar{s}\rangle$ in η_0 will also be increased by a similar amount.

• This could be another source of enhancement for the $B \to K\eta'$ branching ratio.

4 The $B^- \to K^-(\eta, \eta')$ and $B^- \to \pi^-(\eta, \eta')$ decays

• The CKM matrix elements

$$|V_{ub}| = \frac{|V_{cb}V_{cd}^*|}{|V_{ud}^*|}|\sin\beta\sqrt{1+\frac{\cos^2\alpha}{\sin^2\alpha}}. \quad (24)$$

• With $\alpha = (99^{+13}_{-9})^\circ$(PDG) and $|V_{cb}| = (41.78 \pm 0.30 \pm 0.08) \times 10^{-3}$(Barberio), we find

$$|V_{ub}| = 3.60 \times 10^{-3}. \quad (25)$$

in agreement with the exclusive data

$|V_{ub}| = (3.33 - 3.51) \times 10^{-3}$(Barberio).

• The current determination(Abulencia) gives $|V_{td}/V_{ts}| = (0.208^{+0.008}_{-0.006})$ which in turn can be used to determined the angle γ from the unitarity relation:

$$|V_{td}| = \frac{|V_{cb}V_{cd}^*|}{|V_{tb}^*|}|\sin\gamma\sqrt{1+\frac{\cos^2\alpha}{\sin^2\alpha}}. \qquad (26)$$

which gives $\gamma = 66^\circ$ ($|V_{tb}| = 1$) and $\alpha = 91.8^\circ$,in good agreement with the value found in the current UT-fit value of $(88 \pm 16)^\circ$.

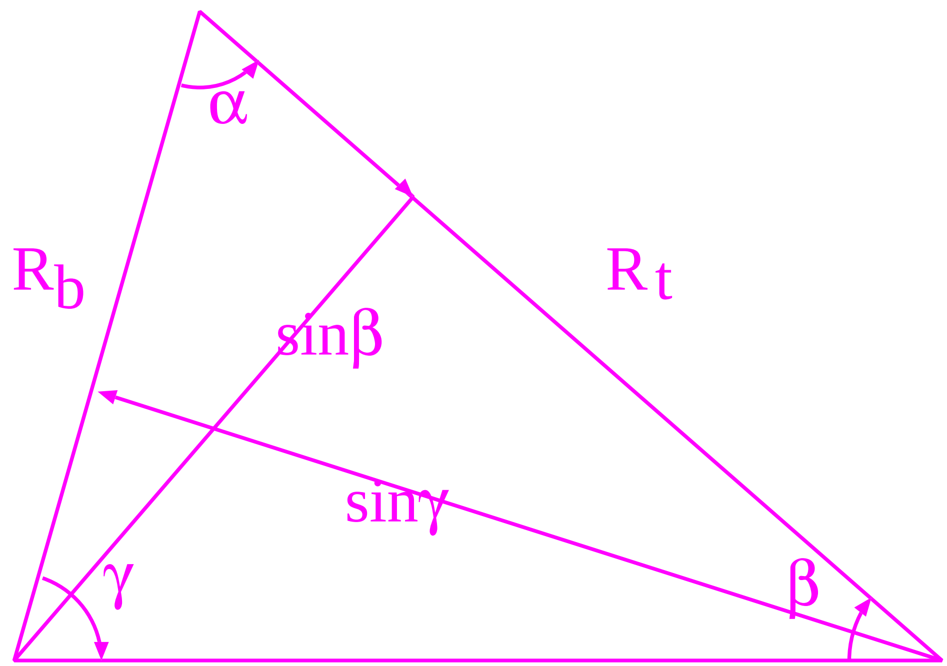

Figure 4: The (db) Unitarity Triangle

- $m_s(2\,\mathrm{GeV}) = 80\,\mathrm{MeV}$, $f_u = f_\pi$, $f_s = f_\pi\left(1 + 2(\frac{f_K}{f_\pi} - 1)\right)$.
- The $B \to \pi$ and $B \to K$ form factor:

$$F_0^{B\pi}(0) = 0.258, \quad F_0^{BK}(0) = 0.33, \text{(Ball and Zwicky)} \tag{27}$$

- From the quark content of η and η',

$$F^{B\eta}(0) = 0.58\, F_0^{B\pi}(0),\ F^{B\eta'}(0) = 0.40\, F_0^{B\pi}(0). \tag{28}$$

and with the s quark content $C_\eta = -0.57$, $C_{\eta'} = 0.82$:

$$\langle 0|\bar{s}\, i\gamma_5 s|\eta\rangle = C_\eta\, B_0 f_s, \quad \langle 0|\bar{s}\, i\gamma_5 s|\eta'\rangle = C_{\eta'}\, B_0 f_s. \tag{29}$$

Decay Modes	QCDF BR ($\times 10^{-6}$)	Experiment
$B^- \to \pi^- \pi^0$	5.05	5.7 ± 0.4
$\bar{B}^0 \to K^- \pi^+$	18.25	19.04 ± 0.6
$B^- \to \pi^- \eta$	3.39	4.4 ± 0.4
$B^- \to \pi^- \eta'$	1.91	$2.6^{+0.6}_{-0.5}$
$B^- \to K^- \eta$	0.43	2.2 ± 0.3
$B^- \to K^- \eta'$	48.26	$69.7^{+2.8}_{-2.7}$

Table 1: The Branching ratios of $B \to P\eta, B \to P\eta'$ in QCDF

• The predicted $\mathcal{B}(B^- \to \pi^- \eta)$ agrees rather well with experiment, but $\mathcal{B}(B^- \to \pi^- \eta')$ is below the Babar value of $(4.0 \pm 0.8 \pm 0.4) \times 10^{-6}$.

• by increasing the $F^{B\to\eta'}$ form factor by $40-50\%$ from the nonet symmetry value, one would get, with $F_0^{B\to\eta'}(0) = 0.156$,

$$\mathcal{B}(B^- \to \pi^- \eta') = 3.89 \times 10^{-6},$$
$$\mathcal{B}(B^- \to K^- \eta') = 61.84 \times 10^{-6}. \qquad (30)$$

which largely improves the prediction for $\mathcal{B}(B^- \to K^- \eta')$.

- Additional source of enhancement of $\mathcal{B}(B^- \to K^- \eta')$ could come from a possible higher order $SU(3)$ breaking effects in the matrix element $\langle 0|\bar{s}\, i\gamma_5\, s|s\bar{s}\rangle$ for η_0.

- With a 15% increase of this matrix element from its nonet value, we would have $\mathcal{B}(B^- \to K^- \eta') = 69.37 \times 10^{-6}$

- The central question is the $F^{B\to\eta'}$ form factor, which need to be determined by a measurement of $\mathcal{B}(B^- \to \eta' \ell \nu)$.

- The new Babar upper limit $\mathcal{B}(B^+ \to \eta' \ell^+ \nu)/\mathcal{B}(B^+ \to \eta \ell^+ \nu) < 0.57$ which is consistent with nonet symmetry for the $B \to \eta, \eta'$ form factors, however shows no evidence for a large value for the $F^{B\to\eta'}$ form factor compared with the usual nonet symmetry value.

5 Conclusion

• We have shown that nonet symmetry for the pseudo-scalar meson mass term implies nonet symmetry for the pseudo-scalar density matrix elements. With this approximate relation, we obtained an improved estimate for the $B \to P\eta'(P = K, \pi\ )$ branching ratios. With a moderate annihilation contribution consistent with the measured $B \to K\pi$ branching ratio, we find that a major part of the $B \to K\eta'$ branching ratio could be obtained by QCDF. Without fine tuning or a large $F^{B\to\eta'}$ form factor, we find that the $B \to K\eta'$ branching ratio is underestimated by $20 - 30\%$. This could be considered as a more or less successful prediction for QCDF, considering the theoretical uncertainties involved.

• This could also indicate that an additional power-suppressed terms could bring the branching ratio close to experiment, as with the $B \to K^{*}\pi$ and $B \to K^{*}\eta$ decay for which the measured branching ratios are much bigger than the QCDF prediction.

6 Acknowledgments

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