FCNC Effects in Type III Seesaw Model

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Introduction

- To accommodate the observation on neutrino masses, the Standard Model must be extended.

  ➔ Strong candidates: models with the “seesaw mechanism.”

- Three types of seesaw models

  -- Type I: introduce singlet right-handed neutrinos ν_R : (1,1,0)

  -- Type II: introduce triplet Higgs Δ : (1,3,1)

  -- Type III: introduce triplet fermions Σ : (1,3,0)

- Interesting features of Type III seesaw model

  -- can have low seesaw scale of ~ TeV to realize leptogenesis

  -- may have detectable effects at LHC

  -- new Flavor-Changing Neutral Current at “tree level”

- To present possible new FCNC interactions in a systematic way (→ Lepton flavor violating processes) in Type III seesaw model.

Type III Seesaw Model

(Foot, Lew, He and Joshi (1989))

In addition to the SM particles, Type III seesaw model consists of SU(2) triplets of fermions with zero hypercharge.

The fermion triplet Σ transforms under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ as (1, 3, 0).

$$\Sigma = \begin{pmatrix} N^0/\sqrt{2} & E^+ \\ E^- & -N^0/\sqrt{2} \end{pmatrix}, \quad \Sigma^c = \begin{pmatrix} N^{0c}/\sqrt{2} & E^{-c} \\ E^{+c} & -N^{0c}/\sqrt{2} \end{pmatrix}$$

right-handed left-handed

The renormalizable Lagrangian involving Σ

$$\mathcal{L} = Tr[\overline{\Sigma} i \not{D} \Sigma] - \frac{1}{2} Tr[\overline{\Sigma} M_\Sigma \Sigma^c + \overline{\Sigma^c} M_\Sigma^* \Sigma] - \tilde{H}^\dagger \overline{\Sigma} \sqrt{2} Y_\Sigma L_L - \overline{L_L} \sqrt{2} Y_\Sigma{}^\dagger \Sigma \tilde{H}$$

$$D_\mu = \partial_\mu - i\sqrt{2} g \begin{pmatrix} W_\mu^3/\sqrt{2} & W_\mu^+ \\ W_\mu^- & -W_\mu^3/\sqrt{2} \end{pmatrix}$$

$$L_L = (\nu_L, e_L^-)^T \qquad \tilde{H} = i\tau_2 H^*$$

The renormalizable Lagrangian involving Σ

$$\mathcal{L} = Tr[\overline{\Sigma} i \not{D} \Sigma] - \frac{1}{2} Tr[\overline{\Sigma} M_\Sigma \Sigma^c + \overline{\Sigma^c} M_\Sigma^* \Sigma] - \tilde{H}^\dagger \overline{\Sigma} \sqrt{2} Y_\Sigma L_L - \overline{L_L} \sqrt{2} Y_\Sigma{}^\dagger \Sigma \tilde{H}$$

$$D_\mu = \partial_\mu - i\sqrt{2} g \begin{pmatrix} W_\mu^3/\sqrt{2} & W_\mu^+ \\ W_\mu^- & -W_\mu^3/\sqrt{2} \end{pmatrix}$$

$$L_L = (\nu_L, e_L^-)^T \qquad \tilde{H} = i\tau_2 H^*$$

Define $$E \equiv E_R^{+c} + E_R^-$$

In terms of the component fields, the Lagrangian becomes

$$\begin{aligned} \mathcal{L} \; = \; & \overline{E} i \not{\partial} E + \overline{N_R^0} i \not{\partial} N_R^0 - \overline{E} M_\Sigma E - \left(\overline{N_R^0} \frac{M_\Sigma}{2} N_R^{0c} \; + \; \text{h.c.} \right) \\ & + \; g \left(W_\mu^+ \overline{N_R^0} \gamma_\mu P_R E + W_\mu^+ \overline{N_R^{0c}} \gamma_\mu P_L E \; + \; \text{h.c.} \right) - g \, W_\mu^3 \overline{E} \gamma_\mu E \\ & - \; \left(\frac{1}{\sqrt{2}} (v + h) \overline{N_R^0} Y_\Sigma \nu_L + (v + h) \overline{E} Y_\Sigma l_L \; + \; \text{h.c.} \right) \end{aligned}$$

with $v \equiv \sqrt{2} \langle \phi^0 \rangle = 246$ GeV

One can easily identify the mass terms:

$$\mathcal{L} \sim -(\overline{\nu_L^c}\ \overline{N^0}) \begin{pmatrix} 0 & {Y_\Sigma}^T v/2\sqrt{2} \\ Y_\Sigma v/2\sqrt{2} & M_\Sigma/2 \end{pmatrix} \begin{pmatrix} \nu_L \\ N^{0c} \end{pmatrix}$$

→ "Seesaw mechanism"

$$\mathcal{L} \sim -(\overline{l_R}\ \overline{E_R}) \begin{pmatrix} m_l & 0 \\ Y_\Sigma v & M_\Sigma \end{pmatrix} \begin{pmatrix} l_L \\ E_L \end{pmatrix}$$

After diagonalizing the mass matrices, one can obtain the light neutrino mass matrix:

$$m_\nu = -\frac{v^2}{2} Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma$$

The relevant Lagrangian in the mass eigen-state basis

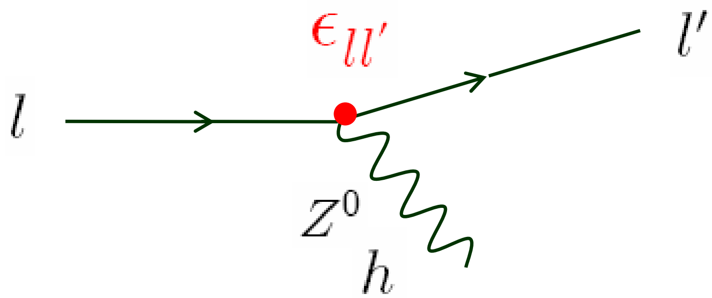
$$\mathcal{L}_{NC} = \frac{g}{cos\theta_W}\bar{l}\gamma^\mu \left(P_L(-\frac{1}{2}+\sin^2\theta_W - \epsilon) + P_R\sin^2\theta_W)\right) l Z_\mu$$
$$\mathcal{L}_H = \frac{g}{2M_W}\bar{l}\,(P_L m_l\,(3\epsilon - 1) + P_R\,(3\epsilon - 1)\,m_l)\,l h\ .$$

$$\epsilon = Y^\dagger M_\Sigma^{-2} Y_\Sigma v^2/2$$

: 3 x 3 matrix

$$\epsilon = \epsilon^\dagger = \begin{pmatrix} \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon^*_{e\mu} & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon^*_{e\tau} & \epsilon^*_{\mu\tau} & \epsilon_{\tau\tau} \end{pmatrix}$$

Non-zero off diagonal elements in ε are the new sources of tree level FCNC in the charged lepton sector.



Classification of FCNC processes

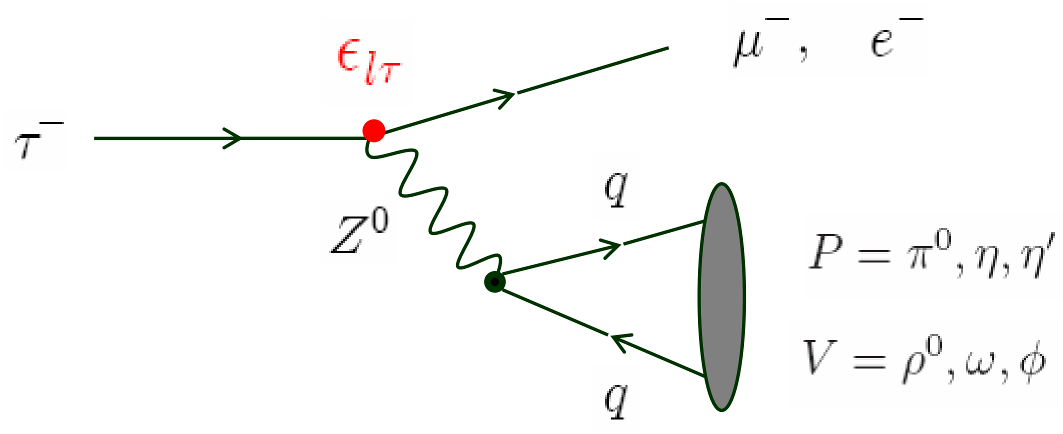
- τ decays to a neutral meson & a charged lepton : $\tau \to Pl$ $\tau \to Vl$
- lepton → 3 leptons : $l_i \to l_j \bar{l}_k l_l$
- radiative decays of a lepton : $l_i \to l_j \gamma$
- μ to e conversion
- muonium-antimuonium oscillation

Lepton flavor violation

- semileptonic 3 body decays of a meson : $M \to M' l \bar{l}'$
- dileptonic decays of a meson : $P \to l \bar{l}'$ $V \to l \bar{l}'$
- dileptonic decays of Z : $Z \to l \bar{l}'$

(Most of them are)
Tree level processes

$\tau \to Pl$ & $\tau \to Vl$ decays



The decay amplitude

$$\mathcal{M} = 2\sqrt{2} G_F \, \epsilon_{l\tau} \sum_{q=u,d,s} \langle M(p_M) | \bar{q} \gamma_\alpha (g_V^q + g_A^q \gamma_5) q | 0 \rangle \cdot \left[\bar{l}(p_l) \gamma^\alpha (1 - \gamma_5) \tau(p_\tau) \right]$$

$$g_V^q = \frac{1}{4} - \frac{2}{3} \sin^2\theta_W \text{ and } g_A^q = -\frac{1}{4} \text{ for up type of quarks}$$

$$g_V^q = -\frac{1}{4} + \frac{1}{3} \sin^2\theta_W \text{ and } g_A^q = \frac{1}{4} \text{ for down type of quarks}$$

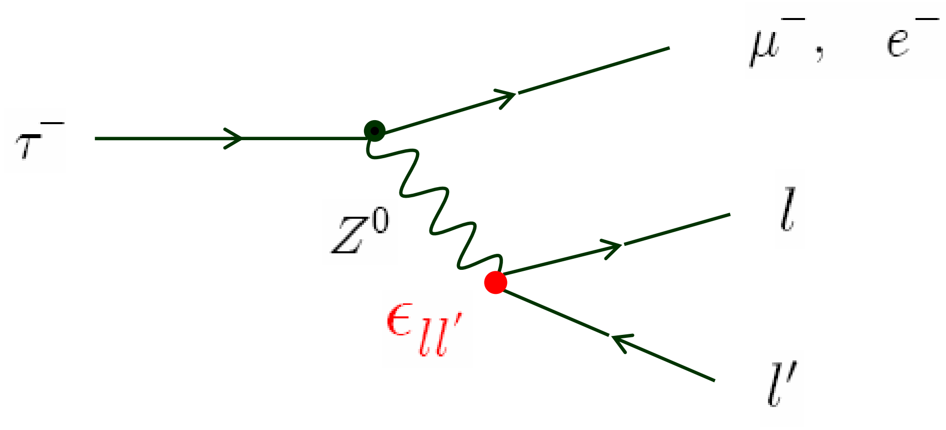
No Higgs contributions: $< P \text{ (or } V) | \bar{q} q | 0 >$ = 0

Process	Branching Ratio	Constraint on $\|\epsilon_{ll'}\|$
$\tau^- \to \pi^0 e^-$	$< 8.0 \times 10^{-8}$	$\|\epsilon_{e\tau}\| < 4.2 \times 10^{-4}$
$\tau^- \to \pi^0 \mu^-$	$< 1.1 \times 10^{-7}$	$\|\epsilon_{\mu\tau}\| < 7.0 \times 10^{-4}$
$\tau^- \to \eta e^-$	$< 9.2 \times 10^{-8}$	$\|\epsilon_{e\tau}\| < 1.2 \times 10^{-3}$
$\tau^- \to \eta \mu^-$	$< 6.5 \times 10^{-8}$	$\|\epsilon_{\mu\tau}\| < 9.7 \times 10^{-4}$
$\tau^- \to \eta' e^-$	$< 1.6 \times 10^{-7}$	$\|\epsilon_{e\tau}\| < 1.0 \times 10^{-3}$
$\tau^- \to \eta' \mu^-$	$< 1.3 \times 10^{-7}$	$\|\epsilon_{\mu\tau}\| < 1.0 \times 10^{-3}$
$\tau^- \to \rho^0 e^-$	$< 6.3 \times 10^{-8}$	$\|\epsilon_{e\tau}\| < 6.5 \times 10^{-4}$
$\tau^- \to \rho^0 \mu^-$	$< 6.8 \times 10^{-8}$	$\|\epsilon_{\mu\tau}\| < 6.8 \times 10^{-4}$
$\tau^- \to \omega e^-$	$< 1.1 \times 10^{-7}$	$\|\epsilon_{e\tau}\| < 3.2 \times 10^{-3}$
$\tau^- \to \omega \mu^-$	$< 8.9 \times 10^{-8}$	$\|\epsilon_{\mu\tau}\| < 2.5 \times 10^{-3}$
$\tau^- \to \phi e^-$	$< 7.3 \times 10^{-8}$	$\|\epsilon_{e\tau}\| < 7.5 \times 10^{-4}$
$\tau^- \to \phi \mu^-$	$< 1.3 \times 10^{-7}$	$\|\epsilon_{\mu\tau}\| < 1.0 \times 10^{-3}$

The most stringent constraint!

- $l \to 3l'$ decays

Abada *et. al.* (2008)



$$\mathrm{Br}(\mu^- \to e^+ e^- e^-) \simeq \frac{\Gamma(\mu^- \to e^+ e^- e^-)}{\Gamma(\mu^- \to e^- \nu_\mu \overline{\nu}_e)}$$

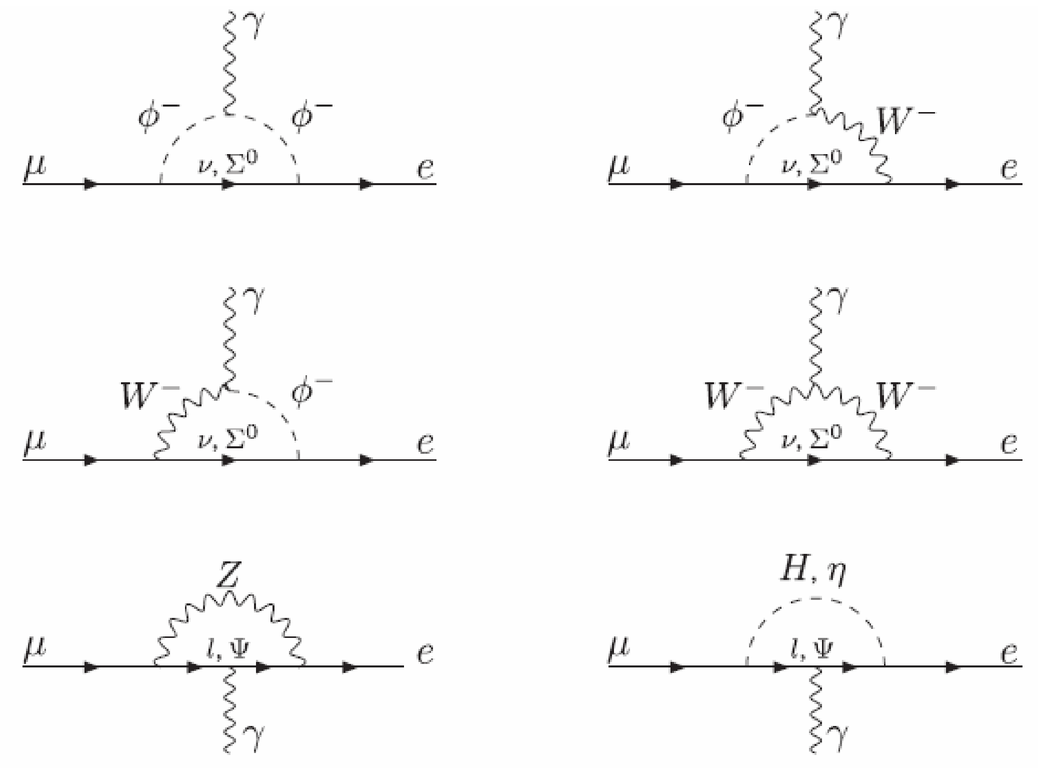
$$= \quad |\epsilon_{e\mu}|^2 \left(3\sin^4\theta_W - 2\sin^2\theta_W + \frac{1}{2}\right)$$

$$\mathrm{Br}(\tau^- \to l_\alpha^+ l_\alpha^- l_\alpha^-) = \frac{\Gamma(\tau^- \to l_\alpha^- l_\alpha^+ l_\alpha^-)}{\Gamma(\tau^- \to e^- \nu_\tau \overline{\nu}_e)} \mathrm{Br}(\tau^- \to e^- \nu_\tau \overline{\nu}_e)$$

$$= \quad |\epsilon_{\alpha\tau}|^2 \left(3\sin^4\theta_W - 2\sin^2\theta_W + \frac{1}{2}\right) \mathrm{Br}(\tau^- \to e^- \nu_\tau \overline{\nu}_e)$$

$\mu \to e\gamma$ & $\tau \to l\gamma$ decays

Abada *et. al.* (2008)



$$\mathrm{Br}(\mu \to e\gamma) = 1.3 \times 10^{-3} \cdot \mathrm{Br}(\mu \to eee)$$

$$\mathrm{Br}(\tau \to \mu\gamma) = 1.3 \times 10^{-3} \cdot \mathrm{Br}(\tau \to \mu\mu\mu)$$

$$\mathrm{Br}(\tau \to e\gamma) = 1.3 \times 10^{-3} \cdot \mathrm{Br}(\tau \to eee)$$

μ to e conversion

Abada *et. al.* (2008)

The relevant diagram is similar to $l \to 3l'$

$$\mathcal{L}_{\text{eff}} = -\sqrt{2} G_F (\bar{l}_i \gamma^\alpha P_L g_{Lij}^{NC} l_j)(\bar{u}\gamma_\alpha[(1 - \tfrac{8}{3}\sin^2\theta_W) - \gamma_5]u$$

$$+ \bar{d}\gamma_\alpha[(-1 + \tfrac{4}{3}\sin^2\theta_W) + \gamma_5]d)$$

$$\text{Br}(\mu \to eee) = 2.4 \times 10^{-1} R^{\mu\to e}$$

$$\text{Br}(\mu \to e\gamma) = 3.1 \times 10^{-4} R^{\mu\to e}$$

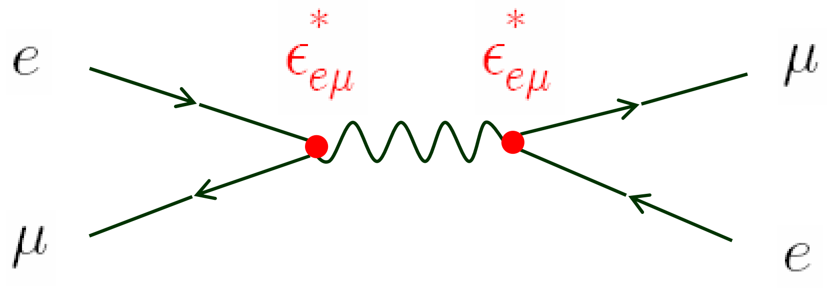
The conversion rate is rather large

➔ provide very strong constraint!

Process	Conversion rate	Constraint on $\lvert\epsilon_{ll'}\rvert$
$\mu - e$ conversion	$< 4.3 \times 10^{-12}$	$\lvert\epsilon_{e\mu}\rvert < 1.7 \times 10^{-7}$
Process	**Branching Ratio**	**Constraint on $\lvert\epsilon_{ll'}\rvert$**
$\mu^- \to e^+e^-e^-$	$< 1 \times 10^{-12}$	$\lvert\epsilon_{e\mu}\rvert < 1.1 \times 10^{-6}$
$\tau^- \to e^+e^-e^-$	$< 3.6 \times 10^{-8}$	$\lvert\epsilon_{e\tau}\rvert < 5.1 \times 10^{-4}$
$\tau^- \to \mu^+\mu^-\mu^-$	$< 3.2 \times 10^{-8}$	$\lvert\epsilon_{\mu\tau}\rvert < 4.9 \times 10^{-4}$
$\tau^- \to \mu^+\mu^-e^-$	$< 4.1 \times 10^{-8}$	$\lvert\epsilon_{e\tau}\rvert < 7.2 \times 10^{-4}$
$\tau^- \to e^+e^-\mu^-$	$< 2.7 \times 10^{-8}$	$\lvert\epsilon_{\mu\tau}\rvert < 5.6 \times 10^{-4}$
$\mu^- \to e\gamma$	$< 1 \times 10^{-15}$	$\lvert\epsilon_{e\mu}\rvert \lesssim 1.1 \times 10^{-4}$
$\tau^- \to e\gamma$	$< 5 \times 10^{-11}$	$\lvert\epsilon_{e\tau}\rvert \lesssim 2.4 \times 10^{-2}$
$\tau^- \to \mu\gamma$	$< 4 \times 10^{-11}$	$\lvert\epsilon_{\mu\tau}\rvert \lesssim 1.5 \times 10^{-2}$

The most stringent constraint !

muonium-antimuonium oscillation



$$H_{eff} = \sqrt{2} G_F \epsilon^{*2}_{e\mu} \bar{\mu}\gamma_\mu(1-\gamma_5)e\bar{\mu}\gamma^\mu(1-\gamma_5)e$$

Probability to observe a transition:

$$P_{\bar{M}M}(0\mathrm{T}) \simeq |\delta|^2/(2\Gamma^2_\mu) \qquad \text{where} \quad \Gamma_\mu \text{ is the muon decay width}$$

$$\delta \equiv 2\langle \bar{M}|H_{eff}|M\rangle \; = 32 G_F \epsilon^2_{e\mu}/(\sqrt{2}\pi a^3) \qquad \text{where } a \simeq (\alpha m_e)^{-1}$$

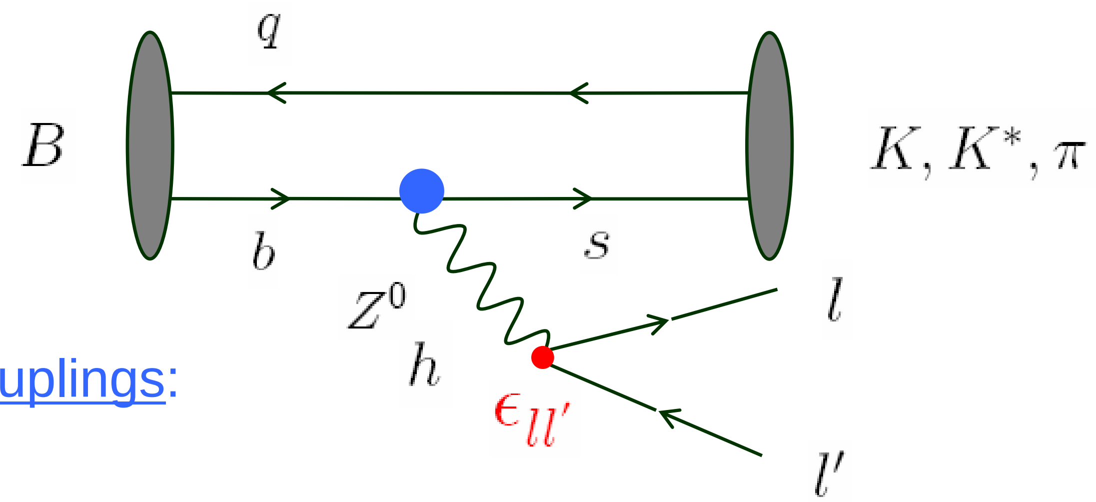
$$P_{\bar{M}M} \leq 8.3 \times 10^{-11} \text{ (90\% C.L.) in 0.1 T magnetic field}$$

$$P_{\bar{M}M}(B) = S_B P_{\bar{M}M}(0\mathrm{T}) \qquad S_B = 0.35$$

$$|\epsilon_{e\mu}| < 4 \times 10^{-2}$$

■ $M \to M' l \bar{l}'$ decays

Semileptonic 3 body decays of B or K



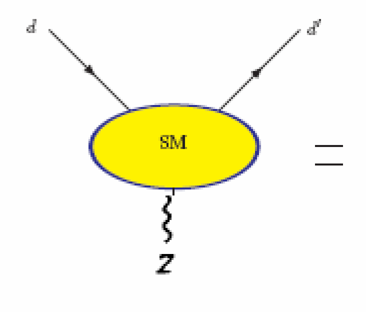
SM effective couplings:

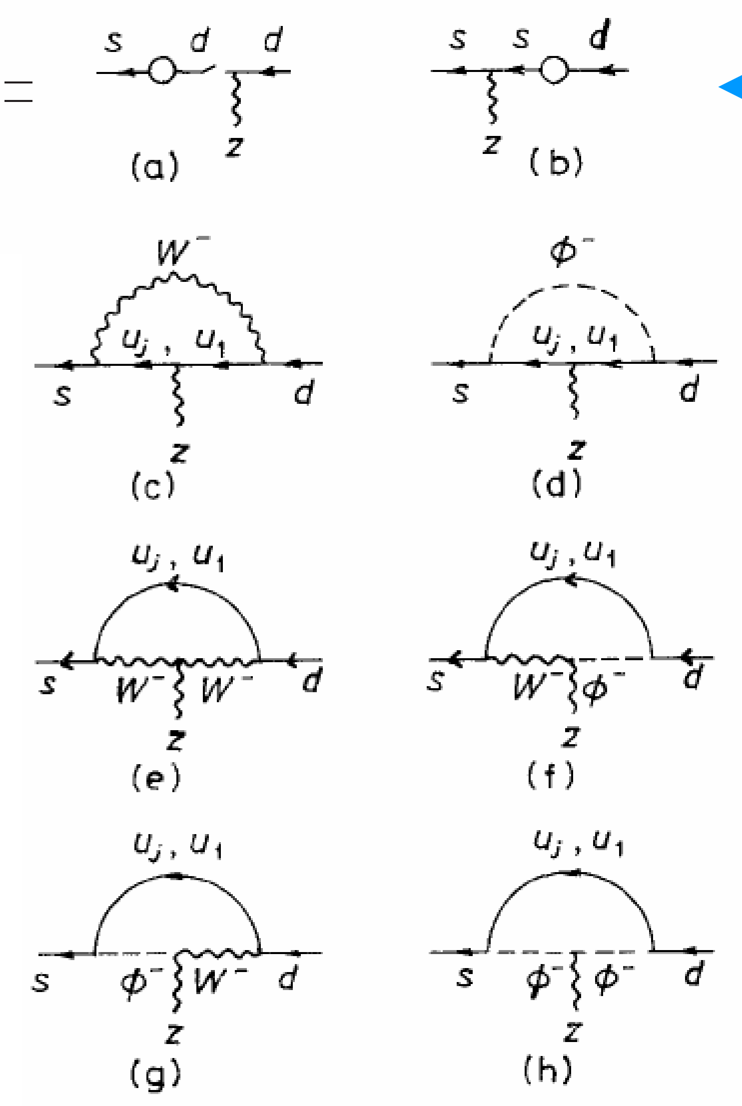
"Z penguins"
&
"Higgs penguins"

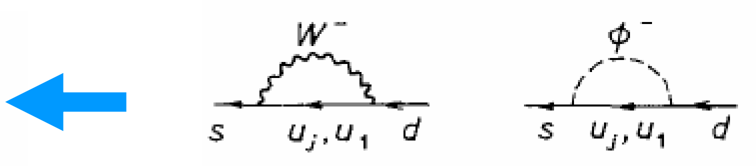
$$\mathcal{L}_{NC} = \frac{g}{cos\theta_W} \bar{l} \gamma^\mu \left(P_L (-\frac{1}{2} + \sin^2\theta_W - \epsilon) + P_R \sin^2\theta_W) \right) l Z_\mu$$

$$\mathcal{L}_H = \frac{g}{2M_W} \bar{l} \left(P_L m_l (3\epsilon - 1) + P_R (3\epsilon - 1) m_l \right) l h \, .$$

“ Z penguins ”





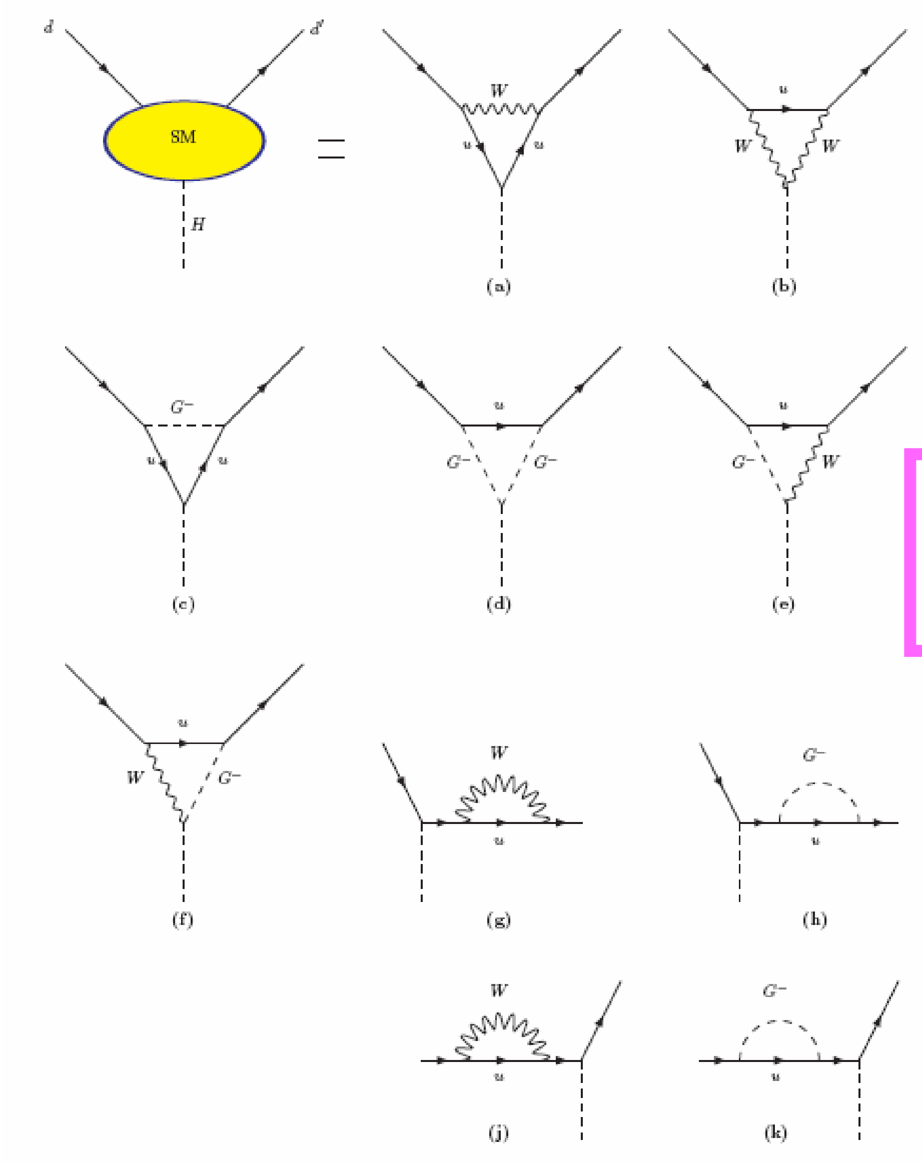


Inami-Lim function

$$C_0(x_i) = \frac{x_i}{8}\left[\frac{x_i - 6}{x_i - 1} + \frac{3x_i + 2}{(x_i - 1)^2}\ln x_i\right]$$

$$x_i = \frac{m_i^2}{m_W^2}$$

“ Higgs penguins ”



$$M(b \to Hs) = \frac{3g^2}{256\pi^2} V_{tb} V_{ts}^* \frac{m_b m_t^2}{m_W^3} \bar{s}(1+\gamma_5) b$$

$$\mathcal{M} = \mathcal{M}^Z + \mathcal{M}^h$$

$$\mathcal{M}^Z = -\frac{1}{32\pi^2} V^*_{iq''} V_{iq'} \frac{g^4}{\cos^2\theta_W M_W^2} C_0(x_i) \epsilon_{ll'} \langle M'(p')|\bar{q}''\gamma_\alpha(1-\gamma_5)q'|M(p)\rangle \times [\bar{u}_l(k_1)\gamma^\alpha(1-\gamma_5)v_{l'}(k_2)] \,,$$

$$\mathcal{M}^h = i\frac{9}{1024\pi^2} V^*_{tq''} V_{tq'} g^4 \frac{m_t^2 m_{q'}}{m_W^4 m_h^2} \epsilon_{ll'} \langle M'(p')|\bar{q}''(1+\gamma_5)q'|M(p)\rangle \times \{\bar{u}_l(k_1)[(m_l + m_{l'}) + (m_{l'} - m_l)\gamma_5]v_{l'}(k_2)\} \,,$$

(i) for $B \to K^{(*)} l\bar{l}'$, $q' = b$ and $q'' = s$

(ii) for $B \to \pi l\bar{l}'$, $q' = b$ and $q'' = d$

(iii) for $K \to \pi l\bar{l}'$, $q' = s$ and $q'' = d$

In the cases of $B \to K^{(*)} l\bar{l}'$ & $K \to \pi l\bar{l}'$:

$|\mathcal{M}^h/\mathcal{M}^Z|$ is suppressed roughly by

$O(x_t(m_b m_l/m_h^2))$ and $O(x_t(m_s m_l/m_h^2))$, respectively.

$$\frac{d\Gamma(B\to Pl\bar{l}')}{dq^2} = \frac{1}{192\pi^5}\frac{G_F^2\alpha^2}{\sin^4\theta_W\cos^4\theta_W}\left|V_{ts}^*V_{tb}\right|^2 C_0^2(x_t)\left|\epsilon_{ll'}\right|^2\frac{\lambda^{3/2}(m_B^2,m_P^2,q^2)}{m_B^3} \times(1-2\rho)^2\left[(1+\rho)\left|f_+(q^2)\right|^2+3\rho\left|f_0(q^2)\right|^2\right],$$

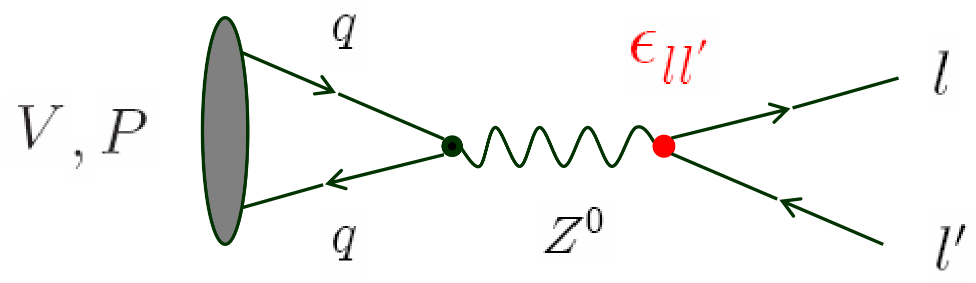
$$\lambda(a,b,c)=(a-b-c)^2-4bc,\ \rho=m_l/(2q^2), \quad f_0(q^2)\equiv\frac{(m_B^2-m_P^2)f_+(q^2)+q^2f_-(q^2)}{\lambda^{1/2}(m_B^2,m_P^2,q^2)}$$

$$\frac{d\Gamma(B\to K^*l\bar{l}')}{ds} = \frac{1}{768\pi^5}\frac{G_F^2\alpha^2}{\sin^4\theta_W\cos^4\theta_W}\left|V_{ts}^*V_{tb}\right|^2 C_0^2(x_t)\left|\epsilon_{ll'}\right|^2 m_B^3\tilde{\lambda}^{1/2} \times\left\{\left|V(q^2)\right|^2\frac{8m_B^4s\tilde{\lambda}}{(m_B+m_{K^*})^2} +\left|A_1(q^2)\right|^2(m_B+m_{K^*})^2\left(\frac{\tilde{\lambda}}{r}+12s\right) +\left|A_2(q^2)\right|^2\frac{m_B^4}{(m_B+m_{K^*})^2}\frac{\tilde{\lambda}^2}{r} -2m_B^2\,\mathrm{Re}\left[A_1(q^2)A_2^*(q^2)\right]\frac{\tilde{\lambda}(1-r-s)}{r}\right\}$$

$$r=m_{K^*}^2/m_B^2,\ s=q^2/m_B^2, \qquad \tilde{\lambda}=1+r^2+s^2-2r-2s-2rs$$

Process	Branching Ratio	Constraint on $\|\epsilon_{ll'}\|$
$B^+ \to \pi^+ e^+ \mu^-$	$< 6.4 \times 10^{-3}$	$\|\epsilon_{e\mu}\| \sim O(1)$
$B^+ \to \pi^+ e^- \mu^+$	$< 6.4 \times 10^{-3}$	$\|\epsilon_{e\mu}\| \sim O(1)$
$B^+ \to \pi^+ e^\pm \mu^\mp$	$< 1.7 \times 10^{-7}$	$\|\epsilon_{e\mu}\| < 0.56$
$B^+ \to K^+ e^+ \mu^-$	$< 9.1 \times 10^{-8}$	$\|\epsilon_{e\mu}\| < 0.18$
$B^+ \to K^+ e^- \mu^+$	$< 1.3 \times 10^{-7}$	$\|\epsilon_{e\mu}\| < 0.21$
$B^+ \to K^+ e^\pm \mu^\mp$	$< 9.1 \times 10^{-8}$	$\|\epsilon_{e\mu}\| < 0.12$
$B^+ \to K^+ \mu^\pm \tau^\mp$	$< 7.7 \times 10^{-5}$	$\|\epsilon_{\mu\tau}\| \sim O(1)$
$B^0 \to \pi^0 e^\pm \mu^\mp$	$< 1.4 \times 10^{-7}$	$\|\epsilon_{e\mu}\| < 0.73$
$B^0 \to K^0 e^\pm \mu^\mp$	$< 2.7 \times 10^{-7}$	$\|\epsilon_{e\mu}\| < 0.21$
$B^+ \to K^*(892)^+ e^+ \mu^-$	$< 1.3 \times 10^{-6}$	$\|\epsilon_{e\mu}\| < 7.1 \times 10^{-2}$
$B^+ \to K^*(892)^+ e^- \mu^+$	$< 9.9 \times 10^{-7}$	$\|\epsilon_{e\mu}\| < 6.2 \times 10^{-2}$
$B^+ \to K^*(892)^+ e^\pm \mu^\mp$	$< 1.4 \times 10^{-7}$	$\|\epsilon_{e\mu}\| < 1.7 \times 10^{-2}$
$B^0 \to K^*(892)^0 e^+ \mu^-$	$< 5.3 \times 10^{-7}$	$\|\epsilon_{e\mu}\| < 4.5 \times 10^{-2}$
$B^0 \to K^*(892)^0 e^- \mu^+$	$< 3.4 \times 10^{-7}$	$\|\epsilon_{e\mu}\| < 3.6 \times 10^{-2}$
$B^0 \to K^*(892)^0 e^\pm \mu^\mp$	$< 5.8 \times 10^{-7}$	$\|\epsilon_{e\mu}\| < 3.4 \times 10^{-2}$
$K^+ \to \pi^+ e^+ \mu^-$	$< 1.3 \times 10^{-11}$	$\|\epsilon_{e\mu}\| < 0.44$ [0.8]
$K^+ \to \pi^+ e^- \mu^+$	$< 5.2 \times 10^{-10}$	$\|\epsilon_{e\mu}\| \sim O(1)$
$K_L \to \pi^0 e^\pm \mu^\mp$	$< 6.2 \times 10^{-9}$	$\|\epsilon_{e\mu}\| \sim O(1)$

$V \to l\bar{l}'$ and $P \to l\bar{l}'$ decays



$V = J/\psi$ or Υ,
$P = \pi^0, \eta$ or η'

$l,\ l' = e,\ \mu,\ \tau\ (l \neq l')$

No Higgs contributions: $< P|\bar{q}q|0 >$ = 0

$$\mathcal{M} = 2\sqrt{2}G_F \epsilon_{ll'} \sum_{q=u,d,s,c,b} \langle 0|\bar{q}\gamma_\alpha(g_V^q + g_A^q\gamma_5)q|M(p_M)\rangle \cdot [\bar{u}_l(p_1)\gamma^\alpha(1-\gamma_5)v_{l'}(p_2)]$$

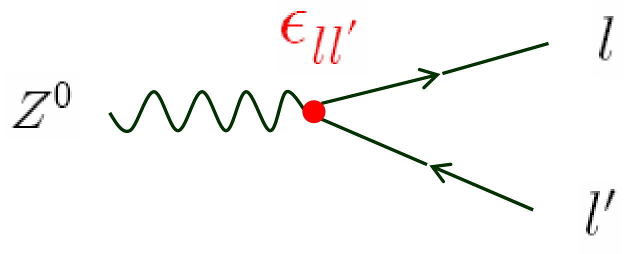
$$g_V^q = \frac{1}{4} - \frac{2}{3}\sin^2\theta_W \qquad g_A^q = -\frac{1}{4} \text{ for up type of quarks}$$

$$g_V^q = -\frac{1}{4} + \frac{1}{3}\sin^2\theta_W \qquad g_A^q = \frac{1}{4} \text{ for down type of quarks}$$

Process	Branching Ratio	Constraint on $\|\epsilon_{ll'}\|$
$\Upsilon(3S) \to e^{\pm}\tau^{\mp}$	$< 5 \times 10^{-6}$	$\|\epsilon_{e\tau}\| < 0.39$
$\Upsilon(3S) \to \mu^{\pm}\tau^{\mp}$	$< 4.1 \times 10^{-6}$	$\|\epsilon_{\mu\tau}\| < 0.35$
$J/\Psi(1S) \to e^{\pm}\mu^{\mp}$	$< 1.1 \times 10^{-6}$	$\|\epsilon_{e\mu}\| \sim O(1)$
$J/\Psi(1S) \to e^{\pm}\tau^{\mp}$	$< 8.3 \times 10^{-6}$	$\|\epsilon_{e\tau}\| \sim O(1)$
$J/\Psi(1S) \to \mu^{\pm}\tau^{\mp}$	$< 2.0 \times 10^{-6}$	$\|\epsilon_{\mu\tau}\| \sim O(1)$
$\pi^0 \to e^+\mu^-$	$< 3.4 \times 10^{-9}$	$\|\epsilon_{e\mu}\| < 0.80$
$\pi^0 \to e^-\mu^+$	$< 3.8 \times 10^{-10}$	$\|\epsilon_{e\mu}\| < 0.27$
$\eta \to e^{\pm}\mu^{\mp}$	$< 6 \times 10^{-6}$	$\|\epsilon_{e\mu}\| \sim O(1)$
$\eta' \to e^{\pm}\mu^{\mp}$	$< 4.7 \times 10^{-4}$	$\|\epsilon_{e\mu}\| \sim O(1)$

provide rather weak constraints!

- $Z \to l\,\bar{l}'$ **decays** $(l,\ l' = e,\ \mu,\ \tau \text{ and } l \neq l')$



The decay amplitude:

$$\mathcal{M} = \frac{g}{2\cos\theta_W}\epsilon_{ll'}\epsilon_\alpha(p)\cdot[\bar{u}_l(p_1)\gamma^\alpha(1-\gamma_5)v_{l'}(p_2)]$$

where ϵ_α is the polarization vector of Z.

The decay rate:

$$\Gamma = \frac{G_F}{3\sqrt{2}\pi}|\epsilon_{ll'}|^2 M_Z^3$$

Process	Branching Ratio	Constraint on $\lvert\epsilon_{ll'}\rvert$
$Z \to e^\pm\mu^\mp$	$< 1.7\times10^{-6}$	$\lvert\epsilon_{e\mu}\rvert < 1.8\times10^{-3}$
$Z \to e^\pm\tau^\mp$	$< 9.8\times10^{-6}$	$\lvert\epsilon_{e\tau}\rvert < 4.3\times10^{-3}$
$Z \to \mu^\pm\tau^\mp$	$< 1.2\times10^{-5}$	$\lvert\epsilon_{\mu\tau}\rvert < 4.8\times10^{-3}$

Summary

- In Type III seesaw model, new FCNC interactions can cause lepton flavor violating processes at tree level.

- The most stringent constraints on FCNC effects:

$$\begin{cases} \tau \to \pi^0 e & \longrightarrow \quad |\epsilon_{e\tau}| < 4.2 \times 10^{-4} \\ \tau \to \mu\mu\bar{\mu} & \longrightarrow \quad |\epsilon_{\mu\tau}| < 4.9 \times 10^{-4} \\ \mu - e \ \text{conversion in atomic nuclei} & \longrightarrow \quad |\epsilon_{e\mu}| < 1.7 \times 10^{-7} \end{cases}$$

$$\epsilon = Y^\dagger M_\Sigma^{-2} Y_\Sigma v^2/2$$

- Future experiments at LHCb, J-PARC, Super-B and ILC can further improve constraints.

Back Up

Diagonalization of the mass matrices: (Abada *et. al.*)

$$\begin{pmatrix} l_{L,R} \\ E_{L,R} \end{pmatrix} = U_{L,R} \begin{pmatrix} l'_{L,R} \\ E'_{L,R} \end{pmatrix} , \quad \begin{pmatrix} \nu_L \\ N^{0c} \end{pmatrix} = U_0 \begin{pmatrix} \nu'_L \\ N'^{0c} \end{pmatrix}$$

$U_{L,R,0}$: (3+3)-by-(3+3) matrices if 3 triplets are present.

$$U_L \equiv \begin{pmatrix} U_{Lll} & U_{LlE} \\ U_{LEl} & U_{LEE} \end{pmatrix} , U_R \equiv \begin{pmatrix} U_{Rll} & U_{RlE} \\ U_{REl} & U_{REE} \end{pmatrix} , U_0 \equiv \begin{pmatrix} U_{0\nu\nu} & U_{0\nu E} \\ U_{0E\nu} & U_{0EE} \end{pmatrix}$$

To order $v^2 M_\Sigma^{-2}$,

$$U_{Lll} = 1 - \epsilon , \quad U_{LlE} = Y_\Sigma^\dagger M_\Sigma^{-1} v , \quad U_{LEl} = -M_\Sigma^{-1} Y_\Sigma v , \quad U_{LEE} = 1 - \epsilon' ,$$

$$U_{Rll} = 1 , \quad U_{RlE} = m_l Y_\Sigma^\dagger M_\Sigma^{-2} v , \quad U_{REl} = -M_\Sigma^{-2} Y_\Sigma m_l v , \quad U_{LEE} = 1 ,$$

$$U_{0\nu\nu} = (1 - \epsilon/2) U_{PMNS} , \quad U_{0\nu E} = Y_\Sigma^\dagger M_\Sigma^{-1} v/\sqrt{2} , \quad U_{0E\nu} = -M_\Sigma^{-1} Y_\Sigma U_{0\nu\nu} v/\sqrt{2} ,$$

$$U_{0EE} = 1 - \epsilon'/2 , \quad \boxed{\epsilon = Y^\dagger M_\Sigma^{-2} Y_\Sigma v^2/2} \quad \epsilon' = M_\Sigma^{-1} Y_\Sigma Y_\Sigma^\dagger M_\Sigma^{-1} v^2/2 .$$

The decay rate

$$\Gamma = a_P \frac{G_F^2 f_P^2}{2\pi m_\tau^2} |\epsilon_{l\tau}|^2 |\vec{p}_l| \left[m_\tau^4 + m_l^4 - 2m_l^2 m_\tau^2 - (m_l^2 + m_\tau^2) m_P^2 \right]$$

$$f_P = f_\pi \text{ with } a_P = 1 \text{ for } \tau^- \to \pi^0 l$$

$$f_P = f_{\eta^{(\prime)}}^s \text{ with } a_P = 1/2 \text{ for } \tau^- \to \eta^{(\prime)} l$$

$$|\vec{p}_l| = \sqrt{(m_\tau^2 + m_P^2 - m_l^2)^2 - 4m_\tau^2 m_P^2}/(2m_\tau)$$

$$\Gamma = a_V \frac{G_F^2 f_V^2 m_V^2}{\pi m_\tau^2} |\epsilon_{l\tau}|^2 |\vec{p}_l| \left[m_\tau^2 + m_l^2 - m_V^2 + \frac{1}{m_V^2}(m_\tau^2 + m_V^2 - m_l^2)(m_\tau^2 - m_V^2 - m_l^2) \right]$$

$$f_V = f_\rho \text{ with } a_V = (1/2 - \sin^2\theta_W)^2 \text{ for } \tau^- \to \rho^0 l$$

$$f_V = f_\omega \text{ with } a_V = (\sin^2\theta_W/3)^2 \text{ for } \tau^- \to \omega l$$

$$f_V = f_\phi \text{ with } a_V = 2(1/4 - \sin^2\theta_W/3)^2 \text{ for } \tau^- \to \phi l$$

For $K \to \pi l \bar{l}'$, one can normalize the branching ratio to $B(K^+ \to \pi^0 e^+ \nu_e)$

$$B(K^+ \to \pi^0 e^+ \nu_e) = (5.08 \pm 0.05)\%$$

$$\frac{B(K^+ \to \pi^+ l \bar{l}')}{B(K^+ \to \pi^0 e^+ \nu_e)} = \frac{2\alpha^2}{\pi^2 \sin^4\theta_W \cos^4\theta_W} \left| \frac{V_{ts}^* V_{td}}{V_{us}} \right|^2 C_0^2(x_t) |\epsilon_{ll'}|^2 ,$$

$$\frac{B(K_L \to \pi^0 l \bar{l}')}{B(K^+ \to \pi^0 e^+ \nu_e)} = \frac{\tau_{K_L}}{\tau_{K^+}} \frac{2\alpha^2}{\pi^2 \sin^4\theta_W \cos^4\theta_W} \left| \mathrm{Im} \left(\frac{V_{ts}^* V_{td}}{V_{us}} \right) \right|^2 C_0^2(x_t) |\epsilon_{ll'}|^2$$

- For $V \to l\bar{l}' \ (V = J/\Psi, \Upsilon)$

$$\Gamma = \frac{8G_F^2 f_V^2}{3\pi}(g_V^q)^2|\epsilon_{ll'}|^2|\vec{p}_l|\left[m_V^2 - \frac{1}{2}m_l - \frac{1}{2}m_{l'} - \frac{1}{2m_V^2}(m_l^2 - m_{l'}^2)^2\right]$$

$$g_V^q = g_V^c \ \text{for} \ V = J/\Psi$$

$$g_V^q = g_V^b \ \text{for} \ V = \Upsilon$$

- For $P \to l\bar{l}' \ (P = \pi^0, \eta, \eta')$

$$\Gamma = a_P \frac{G_F^2 f_P^2}{2\pi m_P}|\epsilon_{ll'}|^2|\vec{p}_l|\left[(m_l^2 + m_{l'}^2)m_P^2 - (m_l^2 - m_{l'}^2)^2\right]$$

$$a_P = 1, \ f_P = f_\pi \ \text{for} \ P = \pi^0$$

$$a_P = 1/2, \ f_P = f_{\eta^{(\prime)}}^s \ \text{for} \ P = \eta^{(\prime)}$$