# Spontaneous Symmetry Breaking and Higgs Physics

## Ling-Fong Li

Carnegie Mellon University

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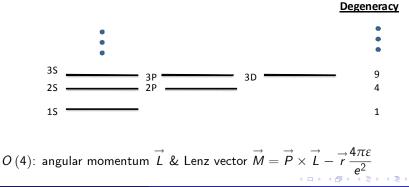
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# Introduction

Symmetry can be realized in 2 different modes: Weyl-Wigner mode

Symmetry of Hamiltonian = Symmetry of States

Example 1:Hydrogen Atom Spectrum

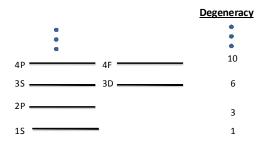


H-Atom

Example 2: Three dimensioal Isotropic Harmonic oscillator

$$H=\hbar\omega\left(a_{1}^{\dagger}a_{1}+a_{2}^{\dagger}a_{2}+a_{3}^{\dagger}a_{3}
ight)+rac{3}{2}\hbar\omega$$

#### 3-d oscillator



Extra degeneracies can be understood with SU(3) symmetry. Nambu-Goldstone mode:

Symmetry of Hamiltonian > Symmetry of States

Symmetry of interaction not realized in states & ground state not invariant 🛓 🕤

Example: Ferromagnetism

 $T > T_C$  (Curie Temp) all magnetic dipoles are randomly oriented  $T < T_C$  all magnetic dipoles are in same direction Ginsburg-Landau theory:

Write free energy u as function of magnetization M,

$$u(\vec{M}) = (\partial_t \vec{M})^2 + \alpha_1 (T) (\vec{M} \cdot \vec{M}) + \alpha_2 (\vec{M} \cdot \vec{M})^2,$$

where

$$\alpha_2 > 0, \qquad \alpha_1 = \alpha \left( T - T_C \right) \qquad \alpha > 0$$

Here u has O(3) rotational symmetry. The ground state is at

$$\vec{M}(\alpha_1 + 2\alpha_2 \vec{M} \cdot \vec{M}) = 0$$

 $T > T_C$  minimum at  $\vec{M} = 0$ .  $T < T_C$  minimum at  $\left| \vec{M} \right| = \sqrt{\alpha_1 / 2\alpha_2} \neq 0$ . If we choose  $\vec{M}$  to be in some direction the rotational symmetry is broken.

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#### **Goldstone Theorem**

Noether theorem: continuous symmetry  $\implies$  conserved current,

$$\partial_{\mu}J^{\mu}=0, \qquad Q=\int d^{3}xJ^{0}\left(x
ight), \qquad rac{dQ}{dt}=0$$

Suppose

$$\left< 0 \left| \left[ Q, \phi \left( 0 
ight) 
ight] 
ight| 0 
ight> = 
u 
eq 0,$$
 and  $u$  indep of time

then

$$\sum_{n} (2\pi)^{3} \delta^{3}(\vec{p}_{n}) \left[ \left\langle 0 \left| J^{0} \right| n \right\rangle \left\langle n \left| \phi \right| 0 \right\rangle e^{-iE_{n}t} - \left\langle 0 \left| \phi \right| n \right\rangle \left\langle n \left| J^{0} \right| 0 \right\rangle e^{iE_{n}t} = v \neq 0$$

To make LHS t independent, we need

$$E_n = 0$$
, as  $\overrightarrow{p}_n = 0$ , for some state  $n$ 

For relativistic system, energy momentum relation,

$$E_n = \sqrt{\stackrel{\rightarrow}{p}_n^2 + m_n^2} \quad \Rightarrow \quad m_n = 0$$
, Goldstone boson

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## Heisenberg Ferromagnets

Hamiltonian is

$$H = -\frac{1}{2} \sum_{RR'} J(\vec{R} - \vec{R}') \vec{S}(\vec{R}) \cdot \vec{S}(\vec{R}')$$

Hamiltonian is symmetric under the rotations of the whole system. Write

$$\vec{S}(\vec{R}) \cdot \vec{S}(\vec{R}') = S_z(\vec{R}) \cdot S_z(\vec{R}') + S_-(\vec{R}) \cdot S_+(\vec{R}')$$

Ground state,

$$|g\rangle = \prod_{R} |S\rangle_{R}$$
,  $S_{z}(\vec{R}) |S\rangle_{R} = S |S\rangle_{R}$ 

all spins aligned in same direction, not invariant under rotation. Excited states:one of the spin is lowered by 1. Define

$$\left| \overrightarrow{R} \right\rangle = \frac{1}{\sqrt{2S}} S_{-}(\overrightarrow{R}) \left| g \right\rangle$$

It turns out that the combination (spin wave)

$$\left| \overrightarrow{k} \right\rangle = \frac{1}{\sqrt{N}} \sum_{k} e^{i \overrightarrow{k} \cdot \overrightarrow{R}} \left| \overrightarrow{R} \right\rangle, \qquad N \text{-total number of spin, } \overrightarrow{k} \text{ arbitrary vector}$$

is an eigenstate of the Hamiltonian with energy,

$$E_k - E_0 = 2S \sum_R J(\vec{R}) \sin^2 \left( \frac{\vec{k} \cdot \vec{R}}{2} \right)$$

which goes to 0 as  $k \rightarrow 0$ , i. e. Goldstone excitations.

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Superfluid Helium atoms Hamiltonian,

$$H = -\frac{1}{2m} \int d^{3}x \psi^{\dagger} \nabla^{2} \psi + \frac{1}{2} \int d^{3}x d^{3}y \psi^{\dagger}(x) \psi^{\dagger}(y) v(x-y) \psi(x) \psi(y)$$

 $v\left(x\right)$  effective potential. System in volume  $\Omega$  . H is invariant under phase transformation,

$$\psi\left(x
ight)
ightarrow\psi'\left(x
ight)=e^{ilpha}\psi\left(x
ight)$$
 ,  $U\left(1
ight)$  symmetry

Conserved charge,

$$Q = \int d^3 x \psi^{\dagger}(x) \psi(x)$$
 number operator

and

$$\left[Q,\psi(x)\right] = \psi(x), \qquad \left[Q,\psi^{\dagger}(x)\right] = -\psi^{\dagger}(x). \tag{1}$$

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In terms of creation and annihilation operators,

$$H = \sum_{k} \frac{\hbar^{2} k^{2}}{2m} a_{k}^{\dagger} a_{k} + \frac{1}{2\Omega} \sum_{k_{i}} \overline{v} (k_{1} - k_{3}) \delta_{k_{1} + k_{2}, k_{3} + k_{4}} a_{k_{1}}^{\dagger} a_{k_{2}}^{\dagger} a_{k_{3}} a_{k_{4}}$$

where

$$\overline{v}(k) = \int d^3x e^{i \overrightarrow{k} \cdot \overrightarrow{x}} v(x)$$

If v(x) is weak, most particles in k = 0 state,

$$< n_0 > \gg < n_k >$$
 with  $k \neq 0$ .

and we can approximate

$$a_0 \simeq a_0^{\dagger} \simeq \sqrt{n_0} = (c - number)$$

To leading order in  $n_{0}$ ,

$$H = \sum_{k \neq 0} \omega_{k} \mathbf{a}_{k}^{\dagger} \mathbf{a}_{k} + \frac{n_{0}}{2\Omega} \sum_{k \neq 0} \overline{\mathbf{v}}\left(k\right) \left(\mathbf{a}_{k}^{\dagger} \mathbf{a}_{-k}^{\dagger} + \mathbf{a}_{k} \mathbf{a}_{-k}\right) + \frac{N^{2}}{2\Omega} \overline{\mathbf{v}}\left(0\right)$$

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where

$$\omega_k = rac{\hbar^2 k^2}{2m} + rac{n_0}{\Omega} \overline{v} \left( k 
ight)$$
, N: total number of particles

H can be diagonalized by Bogoliubov transformation,

$$lpha_k = \operatorname{cosh} heta_k \, \, {\sf a}_k + \operatorname{sinh} heta_k \, \, {\sf a}_{-k}^\dagger$$

$$\alpha_k^{\dagger} = \cosh \theta_k \ a_k^{\dagger} + \sinh \theta_k \ a_{-k}$$

 $\alpha_k$  new quasi-particle operator. With the choice

$$anh 2 heta_k = rac{rac{n_0 v}{\Omega}}{arphi_k}$$

Hamiltonian is diagonal,

$$H = \sum_{k \neq 0} \varepsilon_k \alpha_k^{\dagger} \alpha_k + \frac{N^2 v(0)}{2\Omega} + \frac{1}{2} \sum_{k \neq 0} (\varepsilon_k - \omega_k)$$

where

$$\varepsilon_{k} = \sqrt{\left(\frac{\hbar^{2}k^{2}}{2m}\right)^{2} + 2\left(\frac{\hbar^{2}k^{2}}{2m}\right)\left(\frac{n_{0}\overline{\nu}\left(k\right)}{\Omega}\right)}$$

Energy eigenvalues,

$$E = \sum_{k} n_k \varepsilon_k + \text{const}$$

The quai-particle energy excitation has the property that

 $\varepsilon_k 
ightarrow 0$ , as k 
ightarrow 0 Goldstone excitation

The ground state is

$$\alpha_k | \Psi_0 >= 0 \qquad \forall \, k$$

and  $\psi(0)$  in the ground state is non-zero,  $<\Psi_{0}|\psi(0)|\Psi_{0}>=\sqrt{rac{n_{0}}{\Omega}}
eq 0.$ 

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Spontaneous Symmetry Breaking in Relativistic system 1)Global symmetry Consider

$$L = \frac{1}{2} \left[ \left( \partial_{\mu} \sigma \right)^{2} + \left( \partial_{\mu} \phi \right)^{2} \right] - V \left( \sigma^{2} + \pi^{2} \right)$$

with

$$V\left(\sigma^{2}+\pi^{2}\right)=-\frac{\mu^{2}}{2}\left(\sigma^{2}+\pi^{2}\right)+\frac{\lambda}{4}\left(\sigma^{2}+\pi^{2}\right)^{2}$$

This Lagrangian is invariant under the O(2) rotation

$$\left(\begin{array}{c}\sigma\\\pi\end{array}\right)\longrightarrow \left(\begin{array}{c}\sigma'\\\pi'\end{array}\right)=\left(\begin{array}{c}\cos\alpha&\sin\alpha\\-\sin\alpha&\cos\alpha\end{array}\right)\left(\begin{array}{c}\sigma\\\pi\end{array}\right)$$

Minimize the potential energy V,

$$\frac{\partial V}{\partial \sigma} = \sigma \left[ -\mu^2 + \lambda \left( \sigma^2 + \pi^2 \right) \right] = 0$$
$$\frac{\partial V}{\partial \pi} = \pi \left[ -\mu^2 + \lambda \left( \sigma^2 + \pi^2 \right) \right] = 0$$

For the case  $\mu^2 > 0$ , the minimum is at

$$\sigma^2+\pi^2= {\it v}^2, \qquad {
m with} \ \ {\it v}^2= rac{\mu^2}{\lambda}$$

points on a circle with radius v in the  $(\sigma, \pi)$  plane. They are all related to each other through O(2) rotations and are all equivalent. Pick for example,

$$egin{array}{c|c|c|} 0 \left| \sigma \right| 0 
ight
angle = 
u$$
 ,  $egin{array}{c|c|c|} 0 \left| \pi \right| 0 
ight
angle = 0 \end{array}$ 

O(2) symmetry is broken by the vacuum state.

Consider small oscillations around the true minimum and define a shifted field

$$\sigma' = \sigma - v$$

The Lagrangian is

$$L = \frac{1}{2} \left[ \left( \partial_{\mu} \sigma' \right)^{2} + \left( \partial_{\mu} \phi \right)^{2} \right] - \mu^{2} \sigma'^{2} - \lambda v \sigma' \left( \sigma'^{2} + \pi^{2} \right) - \frac{\lambda}{4} \left( \sigma'^{2} + \pi^{2} \right)^{2}$$

no quadratic term in the  $\pi-{\rm field}\ \Rightarrow\ \pi$  massless Goldstone boson. Note that

massless particle 
$$\implies$$
 long range force

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## Nambu-Jona-Lasinio Model Lagrangina is of the form,

$$L = \overline{\psi} i \gamma^{\mu} \partial_{\mu} \psi + G \left[ \left( \overline{\psi} \psi \right)^{2} - \left( \overline{\psi} \gamma_{5} \psi \right)^{2} \right]$$

Rewrite this as

$$L = \overline{\psi}_{L} i \gamma^{\mu} \partial_{\mu} \psi_{L} + \overline{\psi}_{R} i \gamma^{\mu} \partial_{\mu} \psi_{R} + 4G \left( \overline{\psi}_{R} \psi_{L} \right) \left( \overline{\psi}_{L} \psi_{R} \right)$$

This has chiral  $U(1)_L imes U(1)_R$  symmetry.

To anticipate the spontaneous generateion of fermion mass, split this as

$$L = L_0 + L_{int}$$

with

$$L_0 = \overline{\psi} \left( i \gamma^{\mu} \partial_{\mu} - m \right) \psi, \qquad L_{int} = m \overline{\psi} \psi + G \left[ \left( \overline{\psi} \psi \right)^2 - \left( \overline{\psi} \gamma_5 \psi \right)^2 \right]$$

parameter *m* is arbitrary

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Fermion propagator is ,

$$iS_{F}(p) = rac{i}{p - \Sigma(p, m)}$$

where  $\Sigma(p, m)$  one-particle-irreducible self energy graphs. Take *m* to be the physical mass, then

$$m = \Sigma(p, m)|_{p \neq m}$$

Nambu and Jona-Lasinio take lowest order contribution,

$$\Sigma(p,m)|_{p=m} = 4Gi \int \frac{d^4k}{(2\pi)^4} \frac{m}{k^2 - m^2}$$

Then

$$m = 4Gi \int \frac{d^4k}{\left(2\pi\right)^4} \frac{m}{k^2 - m^2}$$

Certainly, m = 0 is a solution  $\implies$  unbroken symmetry. There is also a non-trivial solution,

$$1 = 4Gi \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2}$$
(2)

where m is non-zero and symmetry is broken spontaneously. Use an ultraviolet cutoff  $\Lambda,$  we get

$$rac{4\pi^2}{G\Lambda^2} = 1 - rac{m^2}{\Lambda^2} \ln\left(rac{\Lambda^2}{m^2} + 1
ight)$$

Since LHS<1, we need

$$G > \frac{4\pi^2}{\Lambda^2}$$
 critical strength (3)

Then Fermion is massive and chiral symmetry is broken spontaneously. It can also be shown that the vacuum expectation value

$$\left\langle ar{\psi}\psi
ight
angle 
eq$$
 0,  $ar{\psi}\gamma_{5}\psi$  — Goldstone mode

Here symmetry breaking is due to the vaccum expectation value of a composite operator, called fermion condensate and Goldstone boson corresponds to a composite operator

QCD at low energies

Approximate chiral  $SU(3) \times SU(3)$  symmetry broken by quark condensate  $\langle \bar{q}q \rangle$  and pseudoscalar meson,  $\pi$ ,  $\kappa$  and  $\eta$  are approxiamte Goldstone bosons.

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# 2) Local Symmetry The Lagrangian for a simple U(1) local symmetry,

$$L = \left(D_{\mu}\phi\right)^{\dagger}\left(D^{\mu}\phi\right) + \mu^{2}\phi^{\dagger}\phi - \lambda\left(\phi^{\dagger}\phi\right)^{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

where

$$\mathcal{D}_\mu \phi = ig(\partial_\mu - i g \mathcal{A}_\muig) \phi, \qquad \mathcal{F}_{\mu
u} = \partial_\mu \mathcal{A}_
u - \partial_
u \mathcal{A}_\mu$$

This is invariant under local U(1) transformation,

$$\phi(x) \longrightarrow \phi'(x) = e^{-i\alpha}\phi(x)$$
$$A_{\mu}(x) \longrightarrow A'_{\mu}(x) = A_{\mu}(x) - \partial_{\mu}\alpha(x)$$

When  $\mu^2 > 0$ , minimum of potential

$$V\left(\phi\right) = -\mu^{2}\phi^{\dagger}\phi + \lambda\left(\phi^{\dagger}\phi\right)^{2}$$

is

$$\phi^{\dagger}\phi = rac{v^2}{2}, \qquad ext{with} \qquad v^2 = rac{\mu^2}{\lambda}$$

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Write

$$\phi = \frac{1}{\sqrt{2}} \left( \phi_1 + i \phi_2 \right)$$

choose

$$\left< 0 \left| \phi_1 \right| 0 \right> = v$$
,  $\left< 0 \left| \phi_2 \right| 0 \right> = 0$ 

 $\Rightarrow \phi_2$  Goldstone boson.

New feature : covariant derivative produces mass term for the gauge boson,

$$\left|D_{\mu}\phi\right|^{2} = \left|\left(\partial_{\mu} - igA_{\mu}\right)\phi\right|^{2} \simeq \dots \frac{g^{2}v^{2}}{2}A^{\mu}A_{\mu} + \dots$$
(4)

Guage boson mass

M = gv

Instead of  $\phi'_1$  and  $\phi'_2$  fields we use  $\eta$  and  $\xi$ . We can now use gauge transformation to transform away  $\xi$ . Define

$$\phi'' = \exp(-i\xi/v)\phi = \frac{1}{\sqrt{2}}[v+\eta(x)]$$
 (5)

and

$$B_{\mu}=A_{\mu}-rac{1}{gv}\partial_{\mu}\xi$$
 (1)

 $\xi$  disappears. In fact  $\xi$  becomes the longitudinal component of  $B_{\mu}$ . massless gauge boson + Goldstone boson= massive vector meson all long range forces disappear.

### Connection with superconductivity

Equation of motion for this theory,

$$\overrightarrow{\nabla} \times \overrightarrow{B} = \overrightarrow{J}$$

with

$$\stackrel{\rightarrow}{J} = ie\left[\phi^{\dagger}\left(\stackrel{\rightarrow}{\nabla} - ie\stackrel{\rightarrow}{A}\right)\phi - \left(\stackrel{\rightarrow}{\nabla} + ie\stackrel{\rightarrow}{A}\right)\phi^{\dagger}\phi
ight]$$

where we have replace g by e.

Spontaneous symmetry breaking  $\Rightarrow \phi = v$  and

$$\overrightarrow{J} = e^2 v^2 \overrightarrow{A}$$

This is just the London equation. Then

$$\vec{\nabla} \times \left( \vec{\nabla} \times \vec{B} \right) = \vec{\nabla} \times \vec{J}, \qquad \Longrightarrow \qquad \nabla^2 \vec{B} = e^2 v^2 \vec{B}$$

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This gives Meissner effect. For the static case,  $\partial_0 \stackrel{\rightarrow}{A} = 0$ ,  $A_0 = 0$ , we get  $\stackrel{\rightarrow}{E} = 0$  and from Ohm's law

$$\stackrel{
ightarrow}{E}=
ho\stackrel{
ightarrow}{J}$$
,  $ho$  resistivity

we have ho= 0, i.e. superconductivity.

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Standard Model of Electroweak Interaction Construction of Standard Model (Weinberg 1967, 't Hooft 1971) This is a gauge theory with spontaneous symmetry breaking.

Gauge group:  $SU(2) \times U(1)$  gauge bosons:  $A_{\mu}, B_{\mu}$ Scalar field:  $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$ , Spontaneous symmetry breaking:  $SU(2) \times U(1) \longrightarrow U(1)_{em}$ 

$$\phi = \exp\left(i\vec{\tau}\cdot\vec{\xi}(x) / v\right) \left(\begin{array}{c}0\\v + H(x)\end{array}\right)$$

Here  $\xi(x)$ , Goldstone bosons eaten up by gauge bosons to become massive. The left over field H(x) is usually called **Higgs Particle**. Massive gauge bosons:

$$\begin{split} W^{\pm}_{\mu} &= \frac{1}{\sqrt{2}} \left( A^{1}_{\mu} \mp i A^{2}_{\mu} \right) & W - \text{boson} \\ Z_{\mu} &= \cos \theta_{W} A^{3}_{\mu} - \sin \theta_{W} B_{\mu} & Z - \text{boson} \\ A_{\mu} &= \sin \theta_{W} A^{3}_{\mu} + \cos \theta_{W} B_{\mu} & Photon \end{split}$$

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Fermions:

a) Leptons

$$L_{i} = \begin{pmatrix} \nu_{e} \\ e \end{pmatrix}_{L}, \begin{pmatrix} \nu_{\mu} \\ \mu \end{pmatrix}_{L}, \begin{pmatrix} \nu_{\tau} \\ \tau \end{pmatrix}_{L}, \qquad R_{i} = e_{R}, \mu_{R}, \tau_{R},$$

b) Quarks (Glashow, Iliopoupos, and Maiani, Kobayashi and Maskawa)

$$q_{iL} = \begin{pmatrix} u' \\ d \end{pmatrix}_{L}, \begin{pmatrix} c' \\ s \end{pmatrix}_{L}, \begin{pmatrix} t' \\ b \end{pmatrix}_{L}, \qquad U_{iR} = u_{R}, c_{R}, t_{R}, \quad D_{iR} = d_{R}, s_{R}, b_{R}$$
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_{L} = U_{u}^{\dagger}U_{d} = U_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Cabbibo-Kobayashi-Maskawa mixing matrix or CKM matrix for short. Yukawa coupling:

$$\mathcal{L}_Y = f_{ij}\overline{L}_iR_j\phi + h.c. + \cdots$$

Fermions get their masses from spontaneous symmetry breaking through Yukawa couplings,

$$m_{ij} = f_{ij}v$$

This implies that the Yukawa couplings  $\propto$  masses.

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Highlights of the Success of Standard Model

- W and Z were discovered in1983 at SPS in CERN,
- Z boson mediates weak neutral current interactions, e.g.

 $u_{\mu} + e \longrightarrow \nu_{\mu} + e, \quad \nu + N \longrightarrow \nu + X, \cdots$ 

These processes were discovered and being studies extensively in 1970's.

- t and b quarks were predicted and subsequently found
- Z bosons are studied extensively in  $e^+e^-$  machine and the results agree with theory

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LHC: look for Higgs,

- *Hf*  $\overline{f}$  coupling  $\propto m_f$  and conserves flavor
- HVV coupling  $\propto M_V$

$$L_{HVV} = gH(x) \left[ M_W W^+_\mu W + \frac{1}{2\cos\theta_W} M_Z Z^\mu Z_\mu \right]$$

Mass of Higgs particle

$$m_{H} = \sqrt{2\mu^{2}} = \sqrt{2\lambda}v,$$
$$v = \sqrt{\frac{\sqrt{2}}{G_{F}}} = 246 \, Gev$$

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## Higgs Mass

- Bound from experimental serach Direct search at LEP  $\Rightarrow M_H > 114.4 \ Gev/c^2$  ([?]). Indirectly, loop correction from high precision electroweak data gives  $76^{+33}_{-24} \ Gev/c^2$ .
- One of the original constraints
  - Pertubative unitarity

Polarization of longituinal W grows with energy.

Lowest order  $W_{L}$  scattering grow with energies and eventually violates the unitarity

 $M_H \leq 870~Gev$ 

This is probably not a true bound but rather a limit on our ability to calculate perturbatively.

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#### O Triviality bound

From the renormaliztion groupe quation, to lowest order, effective quartic Higgs self coupling  $\lambda\left(Q^2\right)$  ,

$$\lambda\left(Q^{2}\right) = \frac{\lambda\left(v^{2}\right)}{1 - \frac{3}{4\pi^{2}}\lambda\left(v^{2}\right)\log\frac{Q^{2}}{v^{2}}}$$

if neglect all other couplings which are small when  $M_H$  large. Taken seriously, ,  $Q^2 \ll v^2$ ,  $\lambda(Q^2) \rightarrow 0$ . We say that the theory is trivial But,  $\lambda(Q^2) \rightarrow \infty$  at

$$Q = \Lambda_{\mathcal{C}} = v \exp\left(rac{2\pi^2}{3\lambda}
ight) = v \exp\left(rac{4\pi^2 v^2}{3M_H^2}
ight)$$

This called Landau pole.

Usual argument : new physics should appear to prevent the instability to develop : the cutoff of Standard Model.

For example  $\Lambda_C \sim 10^{16} \, Gev$  if  $M_H \leq 200$  Gev.

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#### Naturalness problem

Contribution to Higgs self energy diverges quadratically,  $\Sigma_H \sim \Lambda_H^2$  and leads to a shift of Higgs mass,

$$M_H^2=M_{H,0}^2+rac{3\lambda}{16\pi^2}\Lambda_H^2$$

Treat  $\Lambda_H$  as scale where the Standard Model should be cutoff by some unknow new physics.

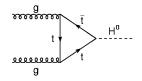
If  $\Lambda_H \sim 10^{19} \text{Gev}$ , then to get  $M_H \leq 1 \text{Tev}$ , fine tune  $M_{H,0}^2$  to 30 decimal places- naturalness problem Supersymmetry-no  $\Lambda_H^2$  term

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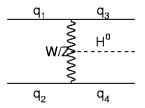
Production of Higgs Paticle

Higgs couplings  $\propto$  masses  $\implies$  production through t quarks, W and Z. :

Gluon fusion through t-quark loop;

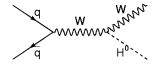


**2** Vector meson fusion;  $qq \rightarrow qqH$ 

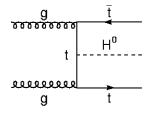


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Solution of H with a gauge boson,  $qq \longrightarrow HW/Z$ 



• Associated Higgs production with heavy quarks  $gg \longrightarrow ttH$ 



The leading order production cross sections for these 4 processes as a function of Higgs boson mass are shown below

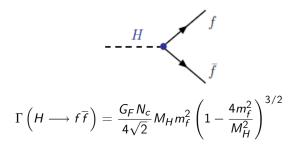
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 $10^2$  $\mathbf{p} + \mathbf{p} \rightarrow \mathbf{H} + \mathbf{X}$  $\sqrt{\mathbf{s}_{\mathbf{pp}}} = 14 \text{ TeV}$  $gg \rightarrow E$ 10<sup>1</sup>  $10^{9}$  $\sigma$  (pb)  $10^{-1}$  $qq \rightarrow qqH$ qq'→ ₩H gg, qq → ttH 10-8 qq → ZH  $10^{-3}$  $10^{-4}$ 200 600 800 400 Ling-Fong Li (Carnegie Mellon University) Spontaneous Symmetry Breaking and Higgs Physics

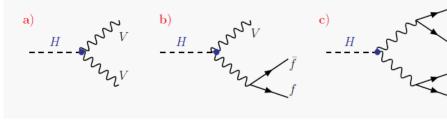
Figure 2.2: The production cross section of the standard model Higgs boson. Across the compl mass range the gluon fusion process is dominating.

Higgs Decay Decay into heaviest particles allowed by kinematics.



wher  $N_c = 3$  for quarks and  $N_c = 1$  for leptons.

 $I \to gauge bosons$ 



 $H \longrightarrow VV$  if  $M_H > 2M_V$ Since longitudinal polarization

$$arepsilon^{\scriptscriptstyle (L)}_{\mu} \simeq rac{k_{\mu}}{M_V} + O\left(rac{M_V}{E}
ight)$$
 , with  $k_{\mu}$  momentum of  $V$ 

for large Higgs masses, the vector bosons are longitudinal polarized.

•  $H \longrightarrow VV^* \longrightarrow Vf\overline{f}$  for  $M_H < 2M_V$ 

For  $M_H \gtrsim 130 \text{ Gev}$ ,  $H \longrightarrow WW \longrightarrow Wf\bar{f}$  dominate over  $H \longrightarrow b\bar{b}$ , because HWW coupling is large enough to compensate the additional coupling constant. and

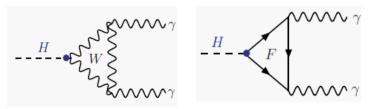
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 $H \longrightarrow V^* V^* \longrightarrow f\overline{f}, ff$ 

These modes is generally small

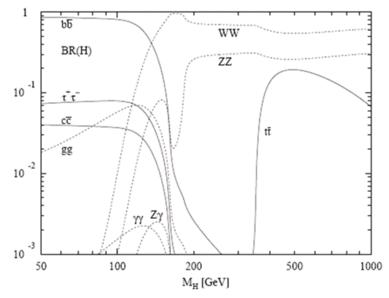
Observe to  $\gamma \gamma$ 

This decay is of special interest due to their relatively clean experimenal signaure.



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branching fractions of various Higgs decay modes are plotted as function of Higgs mass



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