Collective excitations in graphene and the effect of electron-electron interaction

Rafael Roldán

Instituto de Ciencia de Materiales de Madrid CSIC

2nd Workshop on Nanoscience:
Carbon-Related Systems and Nanomaterials

Tainan, July 2012

Introduction

Which are the main features of the excitation spectrum of a two-dimensional (2D) electron?

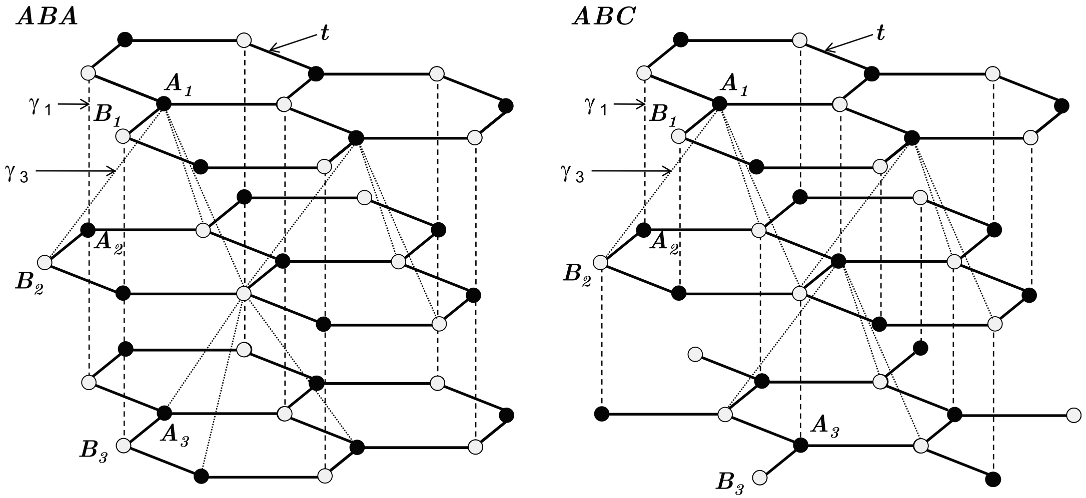
- with/without strong magnetic field: $\mathbf{B} = B\ \hat{\mathbf{z}}$
- with/without electron-electron interactions (RPA): $r_s = \varepsilon_{int}/\varepsilon_F$
- for a standard 2DEG versus doped graphene

	2DEG	(Doped) graphene
$B = 0, r_s = 0$	Continuum PHES	(Intra-) & inter-band continuum PHES
$B = 0, r_s \neq 0$	Continuum PHES + Plasmons	(Intra-) & inter-band continuum PHES + Plasmons
$B \neq 0, r_s = 0$	Discretized PHES (magneto-excitons)	Discretized PHES (hints linear magneto-plasmons)
$B \neq 0, r_s \neq 0$	Discretized PHES (ME) + upper hybrid mode	Discretized PHES (LMP) + upper hybrid mode

PHES = particle-hole excitation spectrum

+ Effect of disorder in the spectrum

π-band tight-binding model for graphene



$$t \approx 2.7 \text{ eV}$$
$$\gamma_1 \approx 0.4 \text{ eV}$$
$$\gamma_3 \approx 0.3 \text{ eV}$$

$$H = \sum_{l=1}^{N_{\text{layer}}} H_l + \sum_{l=1}^{N_{\text{layer}}-1} H'_l$$

$$H_l = -\sum_{<i,j>} (t_{l,ij} a^\dagger_{l,i} b_{l,j} + \text{H.c}) + \sum_i v_{l,i} c^\dagger_{l,i} c_{l,i}$$

$$H'_l = -\gamma_1 \sum_j [a^\dagger_{l,j} b_{l+1,j} + \text{H.c.}]$$
$$- \gamma_3 \sum_{j,j'} [b^\dagger_{l,j} a_{l+1,j'} + \text{H.c.}]$$

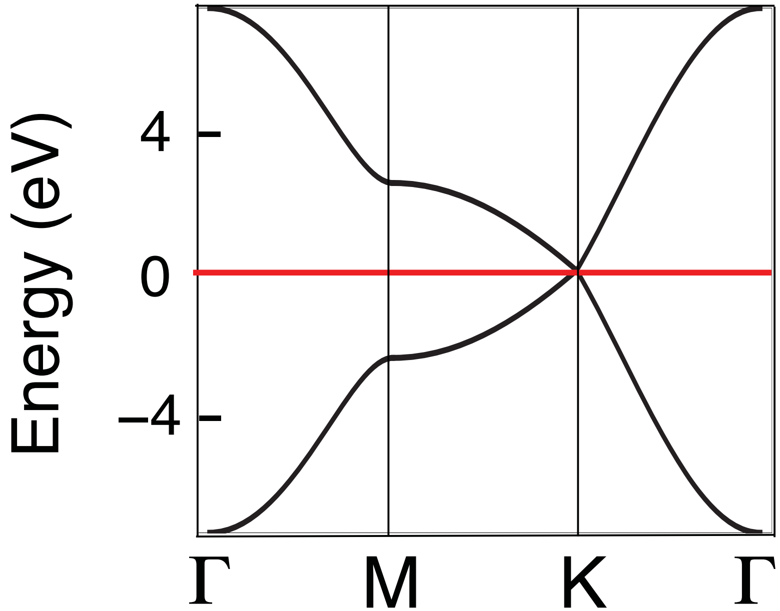
$$H_{\text{imp}} = \varepsilon_d \sum_i d^\dagger_i d_i + V \sum_i (d^\dagger_i c_i + \text{H.c.})$$

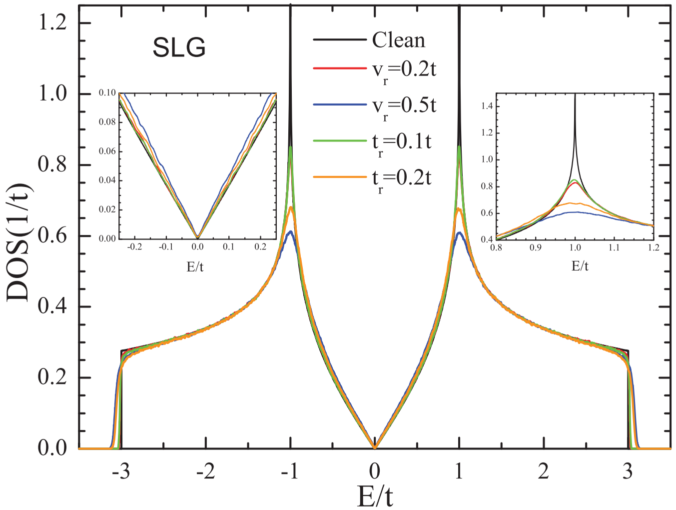
- Peierls substitution accounts for the presence of a magnetic field

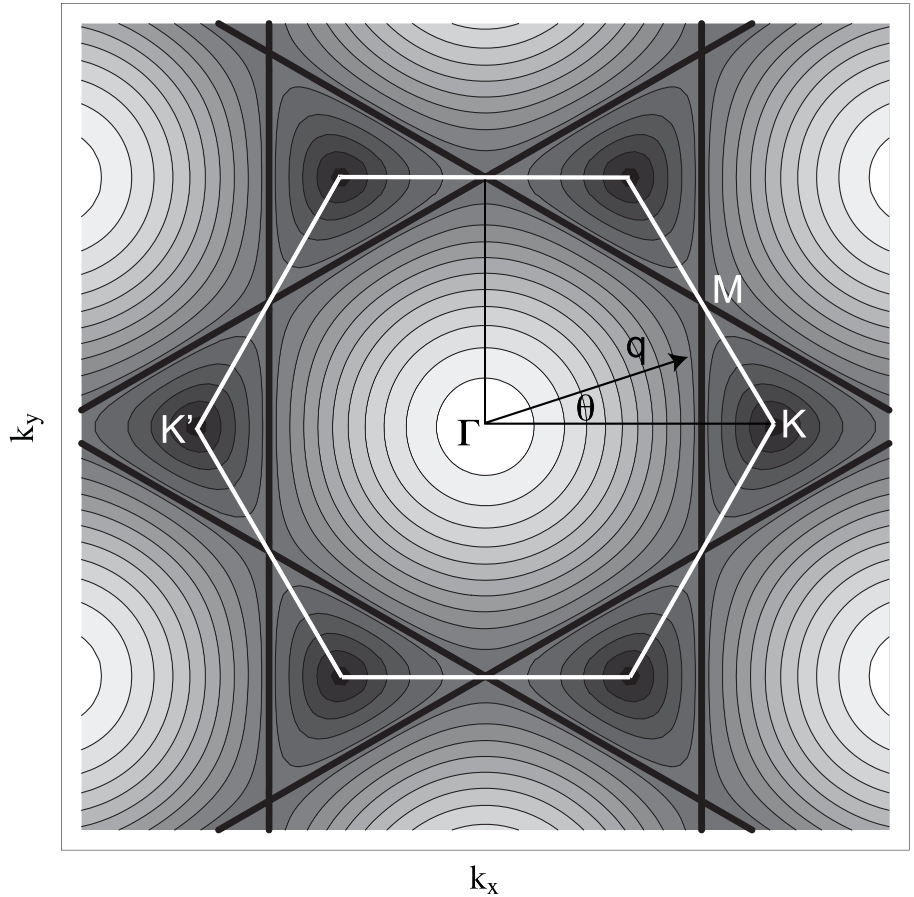
$$t_{ij} \to t_{ij} \exp\left(i \frac{2\pi}{\Phi_0} \int_{\mathbf{R}_i}^{\mathbf{R}_j} \mathbf{A} \cdot d\mathbf{l} \right)$$

$$\Phi_0 = hc/e \qquad \mathbf{A} = (-By, 0, 0)$$

π-band tight-binding model for graphene







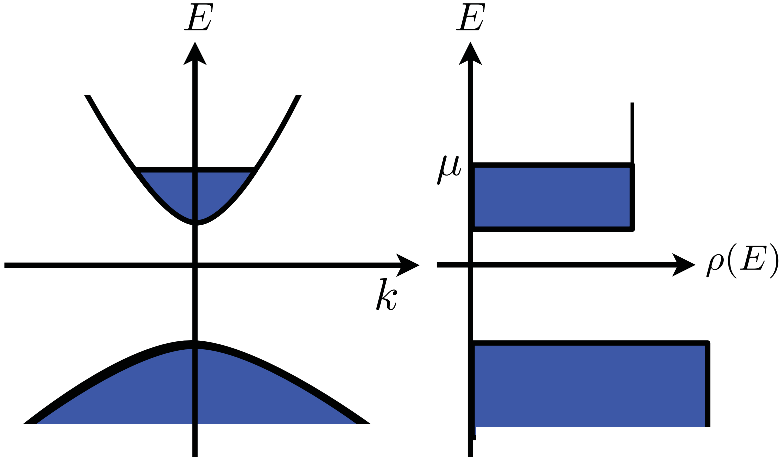
π-band TB dispersion relation

$$\epsilon^0_\lambda(\vec{\mathbf{k}}) = \lambda t \sqrt{1 + 4\cos\left(\frac{3}{2}ak_y\right)\cos\left(\frac{\sqrt{3}}{2}ak_x\right) + 4\cos^2\left(\frac{\sqrt{3}}{2}ak_x\right)}$$

P. R. Wallace Phys. Rev. (1947)

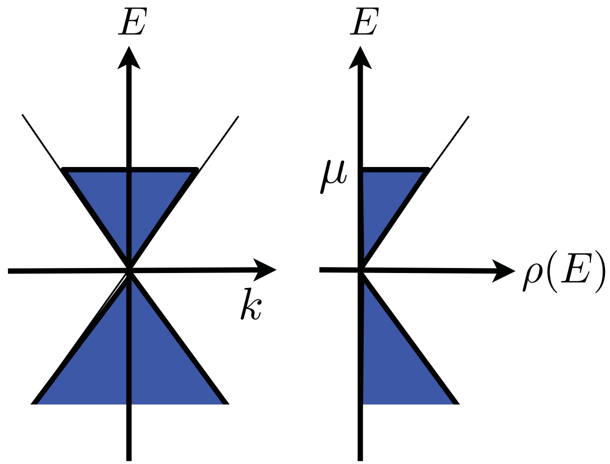
Graphene vs. standard 2DEG: low energy properties

Standard 2DEG



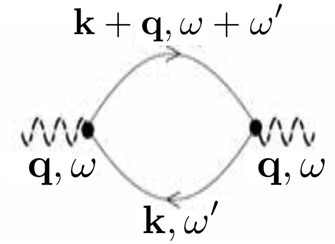
- $\varepsilon_{\mathbf{k}} = \dfrac{\hbar^2 \mathbf{k}^2}{2m_b}$ Quadratic dispersion
- Finite gap (1.4eV for GaAs)
- $\rho(E) = const$ density of states
- Scalar wave-functions

Graphene



- $\varepsilon_\lambda(\mathbf{k}) = \lambda \hbar v_F |\mathbf{k}|$ Linear dispersion
- No gap
- $\rho(E) \propto E$
- Two-component (spinorial) wave-functions

Method: polarization function as a particle-hole pair propagator

$\Pi^0(\mathbf{q},\omega)$ =  = bare particle-hole pair propagator

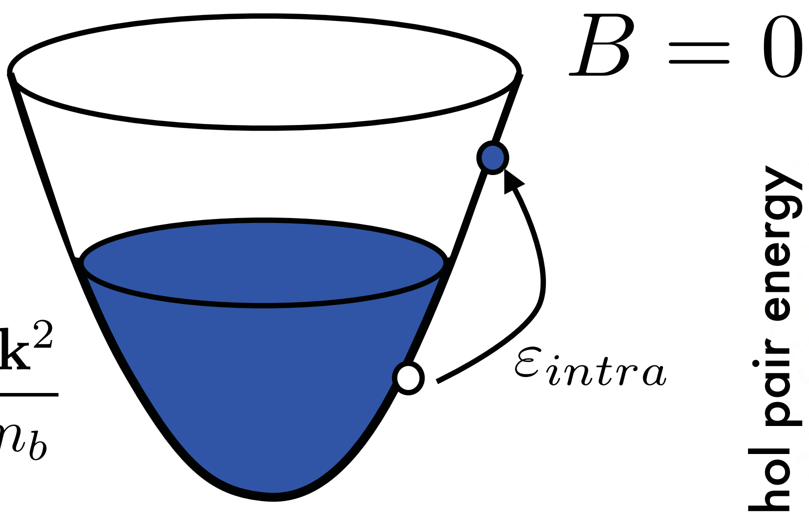
$\Pi(\mathbf{q},\omega)$ = dressed particle-hole pair propagator (e.g. in the RPA)

$S(\mathbf{q},\omega) = -\mathrm{Im}\,\Pi(\mathbf{q},\omega)/\pi$ = spectral density of this propagator (a.k.a. dynamic structure factor)

Pole of $\Pi(\mathbf{q},\omega)$ = collective mode (coherent particle-hole excitation)
$\Rightarrow$ peak in $S(\mathbf{q},\omega)$:
whose position gives the dispersion relation of the collective mode
whose width gives the damping (1/lifetime)

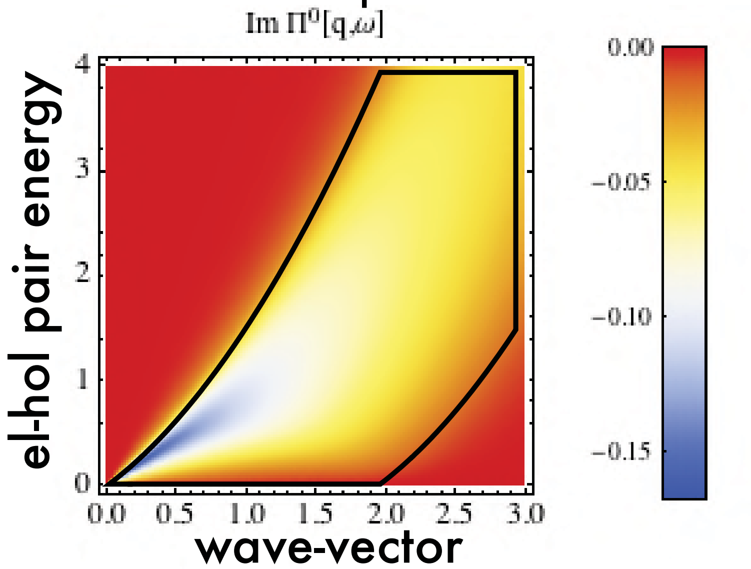
Therefore: we will look for peaks in $S(\mathbf{q},\omega)$

Particle-hole excitations in a standard 2DEG



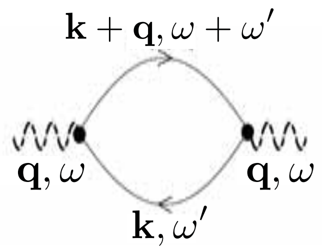
$$\varepsilon_{\mathbf{k}} = \frac{\hbar^2 \mathbf{k}^2}{2m_b}$$

Particle-hole excitation spectrum



The PHES is defined as the (ω, q) region of non-zero spectral weight

$$S(\mathbf{q},\omega) = -\frac{1}{\pi}\mathrm{Im}\Pi(\mathbf{q},\omega)$$



Particle-hole polarization:

$$i\Pi^0(\mathbf{q},\omega) = \int \frac{d\omega' d\mathbf{k}}{(2\pi)^3}\left[G^0(\mathbf{k},\omega')G^0(\mathbf{k}+\mathbf{q},\omega'+\omega)\right]$$

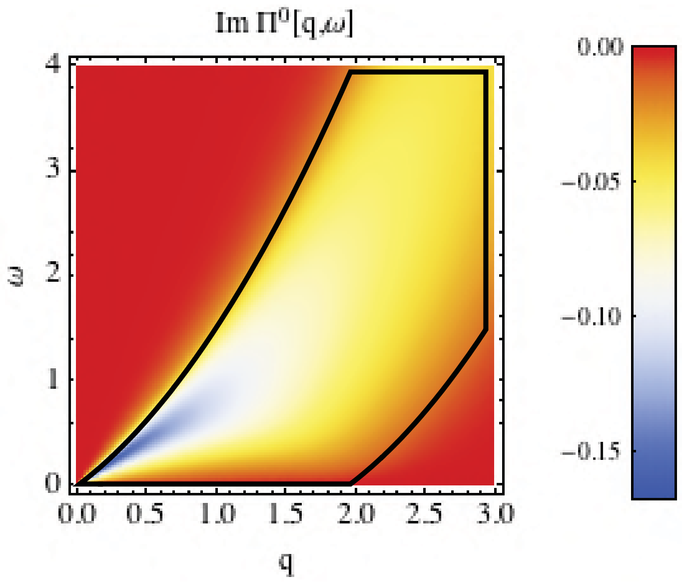
Interacting 2DEG: RPA theory

In RPA the electron-electron interaction is treated self-consistently (sum of bubbles)

$$\Pi^{RPA}(\mathbf{q},\omega) = \frac{\Pi^0(\mathbf{q},\omega)}{1 - v(\mathbf{q})\Pi^0(\mathbf{q},\omega)}$$

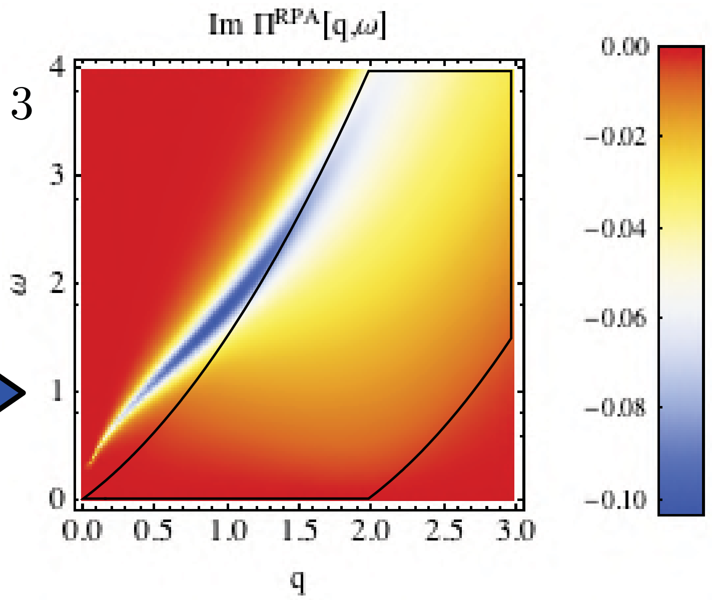
$$B = 0$$

2D Fourier transform of unscreened Coulomb potential $v(\mathbf{q}) = \frac{2\pi e^2}{\epsilon_b |\mathbf{q}|}$



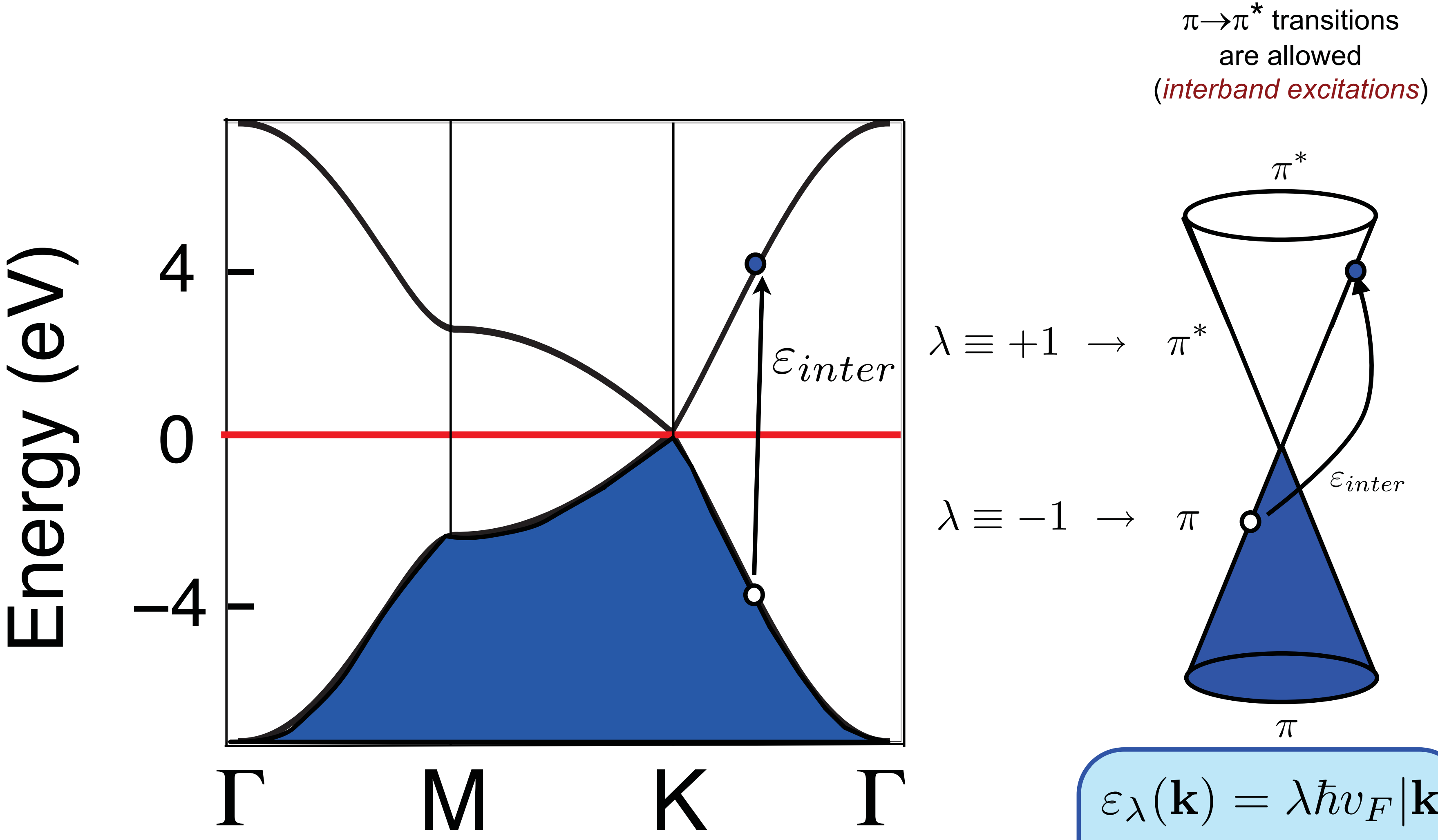
$$r_s \equiv \frac{2m_b e^2}{\hbar^2 \varepsilon_b k_F} \approx 3$$

el-el interactions



The plasmon mode emerges from the continuum when (long-range) interactions between electrons are included

Particle-hole excitations in undoped graphene



$$\varepsilon_\lambda(\mathbf{k}) = \lambda \hbar v_F |\mathbf{k}|$$

$\lambda \equiv$ Band index

PHES of undoped graphene in the Dirac cone approximation

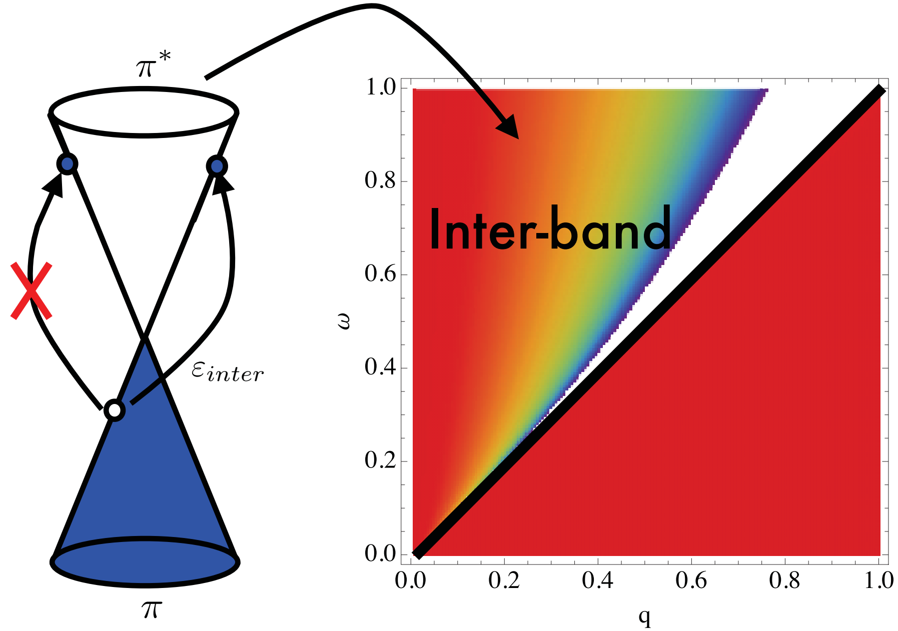
$$\Pi^0(\mathbf{q},\omega) = -\frac{g_s g_v}{(2\pi)^2}\sum_{\lambda,\lambda'=\pm}\int d^2k \frac{n_F[\varepsilon_\lambda(\mathbf{k})] - n_F[\varepsilon_{\lambda'}(\mathbf{k}+\mathbf{q})]}{\omega + \varepsilon_\lambda(\mathbf{k}) - \varepsilon_{\lambda'}(\mathbf{k}+\mathbf{q})) + i\delta} F_{\lambda\lambda'}(\mathbf{k},\mathbf{q})$$

Chirality factor
(wave-function overlap)

$$F_{\lambda,\lambda'}(\mathbf{k},\mathbf{q}) = \frac{1+\lambda\lambda'\cos\theta}{2}$$

$$\Pi^{(1)}(q,\omega) = -\frac{1}{4}\frac{q^2}{\sqrt{v^2q^2-\omega^2}}$$

Concentration around
$\omega = v_F q$



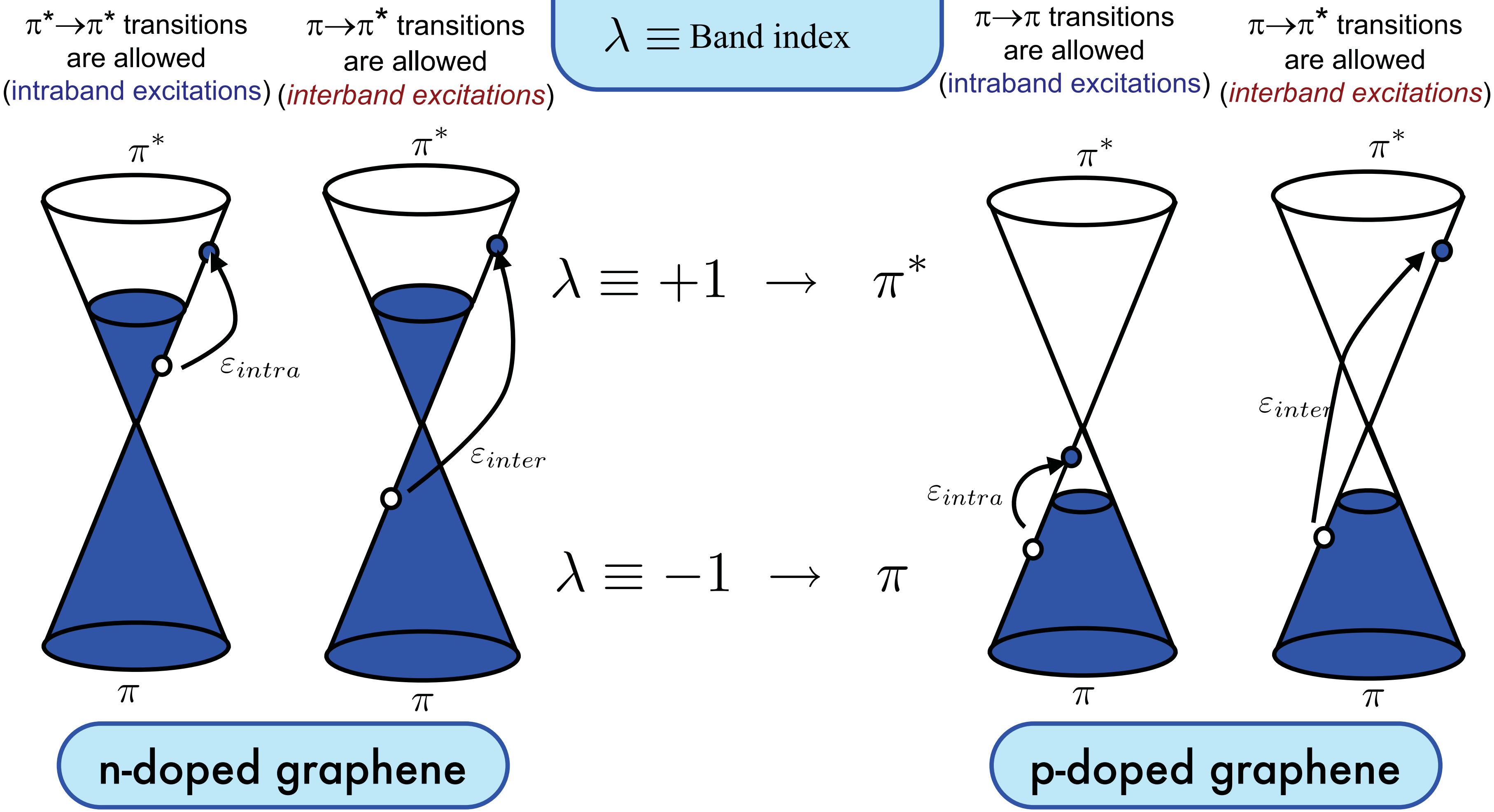
No phase space for *classical* plasmon

see e.g. J. González, F. Guinea & M. A. H. Vozmediano. Nucl. Phys. B 424, 595 (1994)

Particle-hole excitations in doped graphene

$$\varepsilon_\lambda(\mathbf{k}) = \lambda \hbar v_F |\mathbf{k}|$$

$\lambda \equiv$ Band index



PHES in doped graphene: B=0

Particle-hole polarization function for graphene

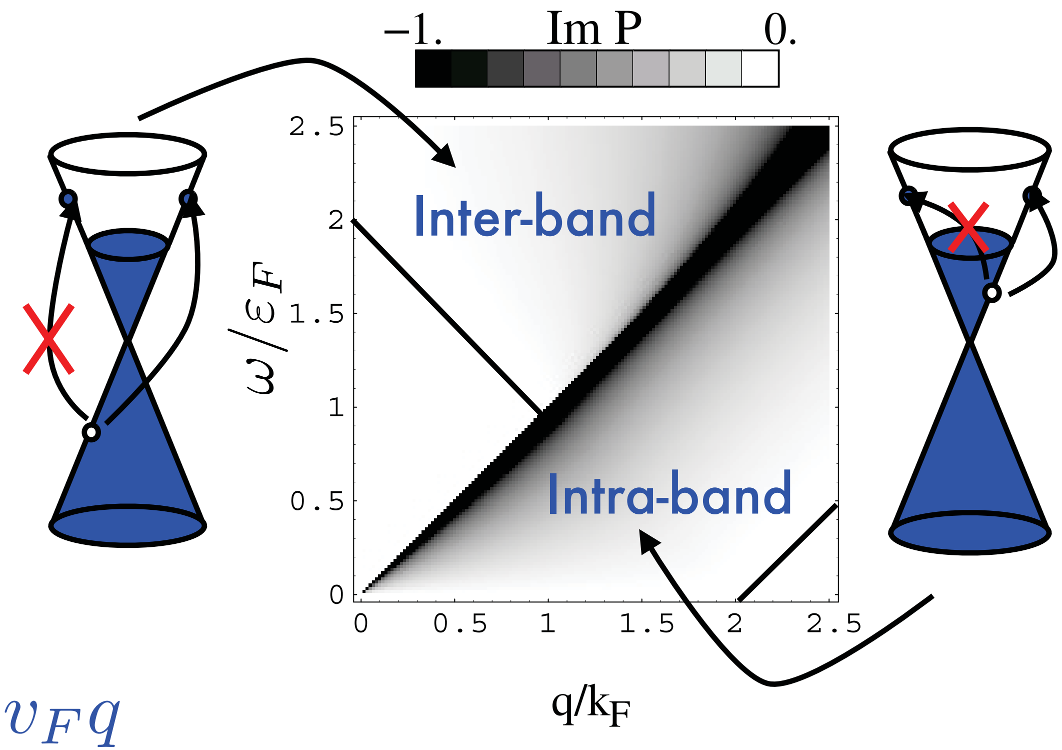
$$\Pi^0(\mathbf{q},\omega) = -\frac{g_s g_v}{(2\pi)^2}\sum_{\lambda,\lambda'=\pm}\int d^2k\,\frac{n_F[\varepsilon_\lambda(\mathbf{k})] - n_F[\varepsilon_{\lambda'}(\mathbf{k}+\mathbf{q})]}{\omega + \varepsilon_\lambda(\mathbf{k}) - \varepsilon_{\lambda'}(\mathbf{k}+\mathbf{q})) + i\delta}F_{\lambda\lambda'}(\mathbf{k},\mathbf{q})$$

Chirality factor (wave-function overlap)

$$F_{\lambda,\lambda'}(\mathbf{k},\mathbf{q}) = \frac{1+\lambda\lambda'\cos\theta}{2}$$

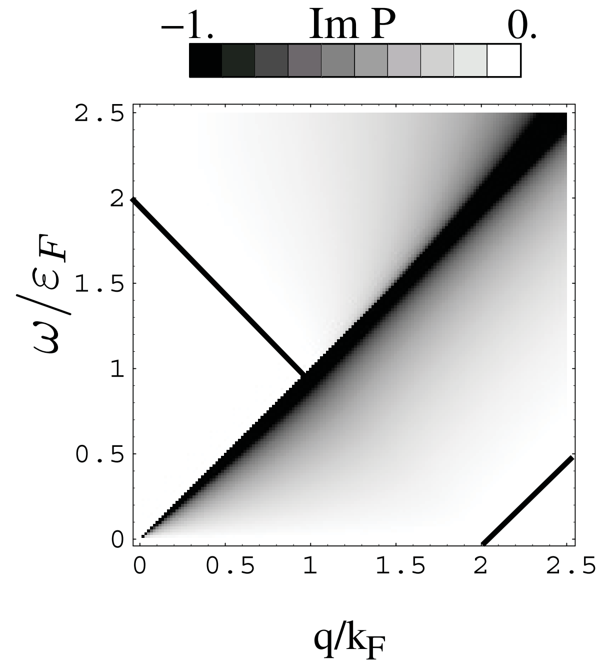
Absence of backscattering

Concentration around $\omega = v_F q$

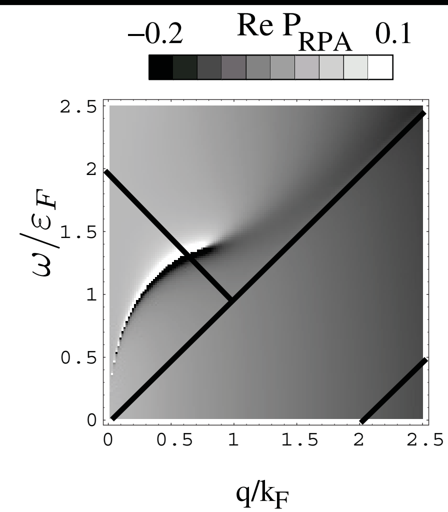


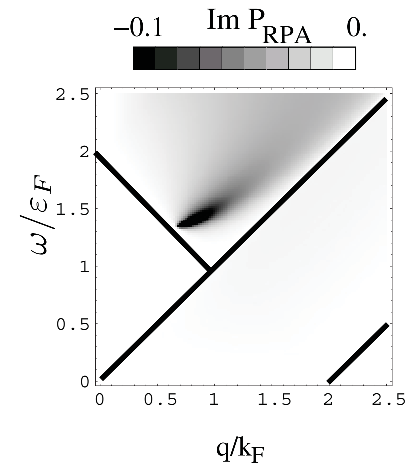
see e.g. B. Wunsch, T. Stauber, F. Sols & F. Guinea. New Journal of Physics 8, 318 (2006)

Interacting graphene at B = 0: RPA theory



el-el interactions





- Interactions lead to a transfer of spectral weight of the PHES to the plasmon mode
- Landau damping when the plasmon mode enters the inter-band region of the PHES

B. Wunsch, T. Stauber, F. Sols & F. Guinea. New Journal of Physics 8, 318 (2006)

PHES of undoped graphene beyond the Dirac cone approximation

Kubo Formula

$$\Pi(\mathbf{q},\omega) = \frac{i}{V}\int_0^\infty d\tau e^{i\omega\tau}\langle[\rho(\mathbf{q},\tau),\rho(-\mathbf{q},0)]\rangle$$

$$\rho(\mathbf{q}) = \sum_i c_i^\dagger c_i \exp(i\mathbf{q}\cdot\mathbf{r}_i)$$

$$\rho(\mathbf{q},\tau) = e^{iH\tau}\rho(\mathbf{q})e^{-iH\tau}$$

$$\Pi(\mathbf{q},\omega) = -\frac{2}{V}\int_0^\infty d\tau e^{i\omega\tau}\mathrm{Im}\langle\varphi|n_F(H)e^{iH\tau} \times \rho(\mathbf{q})e^{-iH\tau}[1-n_F(H)]\rho(-\mathbf{q})|\varphi\rangle$$

$$|\varphi\rangle = \sum_i a_i c_i^\dagger|0\rangle \qquad \textstyle\sum_i |a_i|^2 = 1$$

Lindhard Function

$$\Pi(\mathbf{q},\omega) = -\frac{g_s}{(2\pi)^2}\int_{\mathrm{BZ}} d^2\mathbf{k}\sum_{s,s'=\pm} f_{s\cdot s'}(\mathbf{k},\mathbf{q})\frac{n_F[E^s(\mathbf{k})]-n_F[E^{s'}(\mathbf{k}+\mathbf{q})]}{E^s(\mathbf{k})-E^{s'}(\mathbf{k}+\mathbf{q})+\omega+i\delta}$$

$$E^\pm(\mathbf{k}) = \pm t|\phi_\mathbf{k}| - \mu$$

$$\phi_\mathbf{k} = 1 + 2e^{i3k_xa/2}\cos\left(\frac{\sqrt{3}}{2}k_ya\right)$$

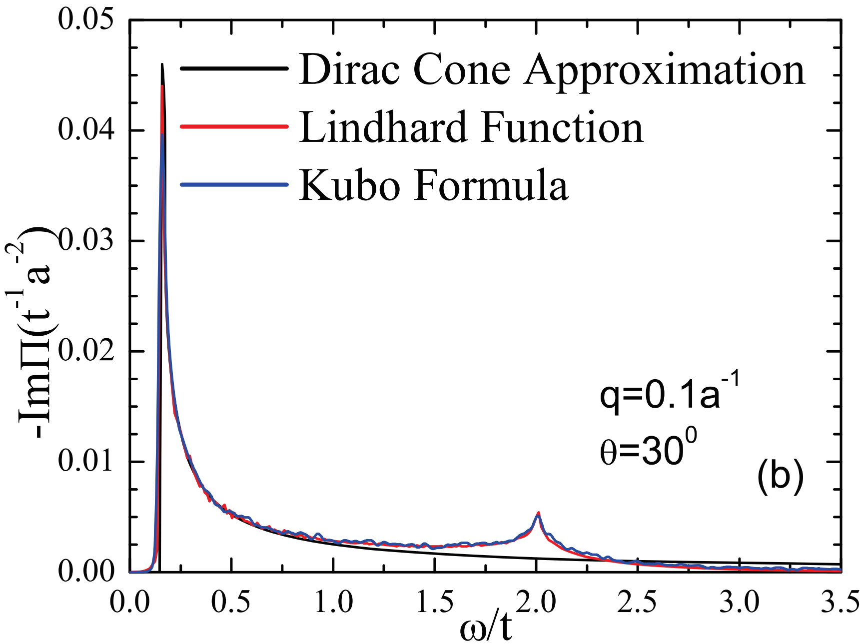
$$f_\pm(\mathbf{k},\mathbf{q}) = \frac{1}{2}\left(1 \pm \mathrm{Re}\left[e^{iq_xa}\frac{\phi_\mathbf{k}}{|\phi_\mathbf{k}|}\frac{\phi^*_{\mathbf{k}+\mathbf{q}}}{|\phi_{\mathbf{k}+\mathbf{q}}|}\right]\right)$$

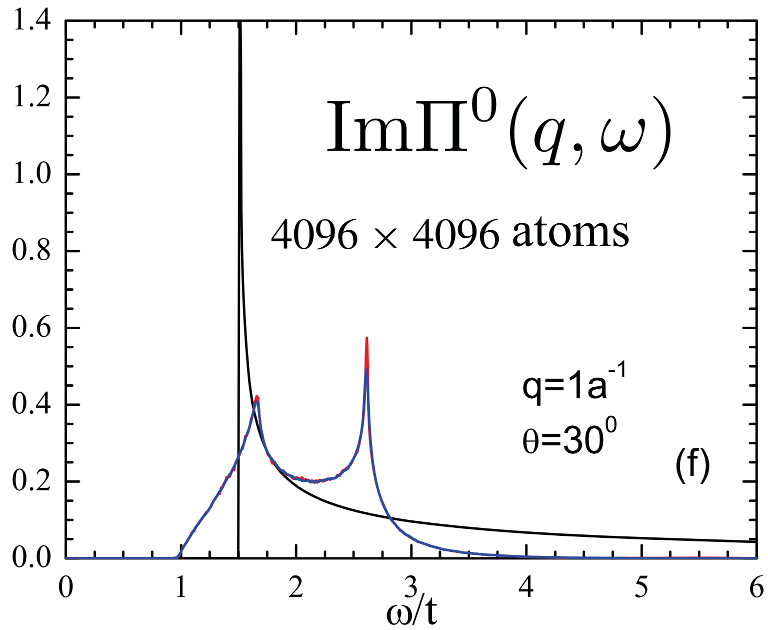
see also T. Stauber, J. Shliemann & N. M. R. Peres Phys. Rev. B **81**, 085409 (2010)

S. Yuan, H. De Raedt & M. I. Katsnelson. Phys. Rev. B **82**, 115448 (2010)

S. Yuan, RR & M. I. Katsnelson. Phys. Rev. B **84**, 035439 (2011)

PHES of undoped graphene beyond the Dirac cone approximation

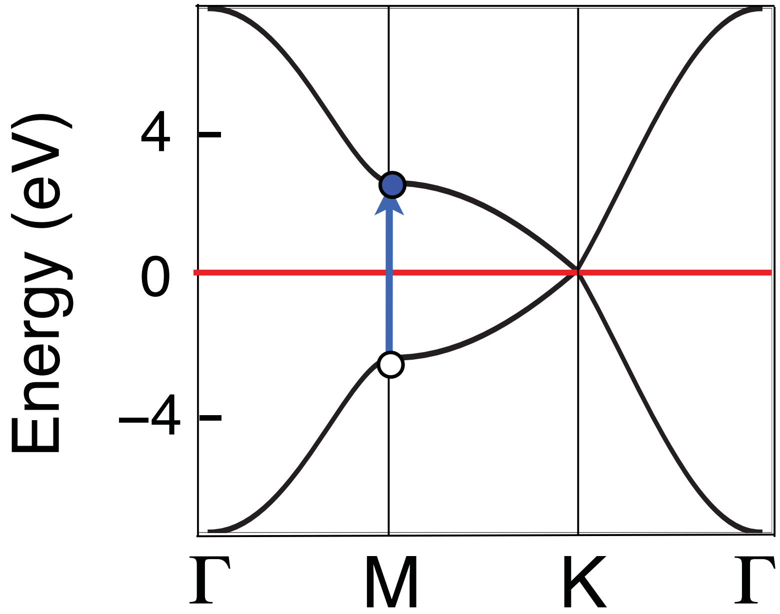


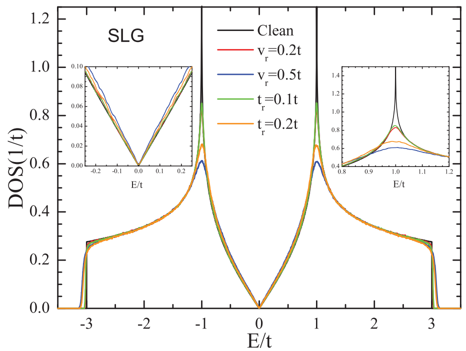


- Excelent agreement between Kubo formula and Lindhard function results
- Dirac cone approximation only valid at low energies

S. Yuan, RR & M. I. Katsnelson. Phys. Rev. B **84**, 035439 (2011)

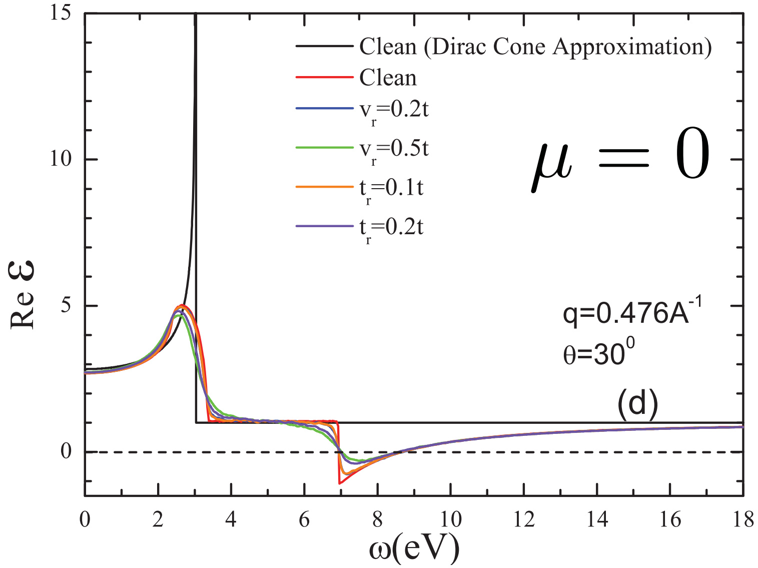
High energy π-Plasmons





The van Hove singularities leads to peaks in the DOS and to zeros of the dielectric function…

$$\mathrm{Re}\varepsilon(\mathbf{q},\omega_{pl}) = 1 - V(q)\mathrm{Re}\Pi(\mathbf{q},\omega_{pl}) = 0.$$



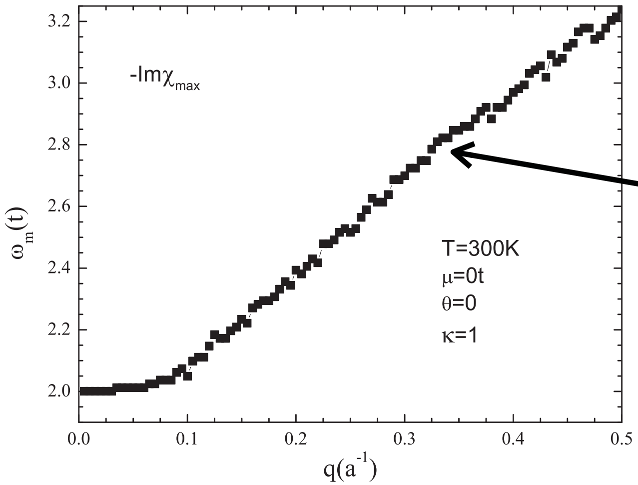
…but these modes are damped

$$\gamma = \frac{\mathrm{Im}\Pi(\mathbf{q},\omega_{pl})}{\frac{\partial}{\partial\omega}\mathrm{Re}\Pi(\mathbf{q},\omega)|_{\omega=\omega_{pl}}}$$

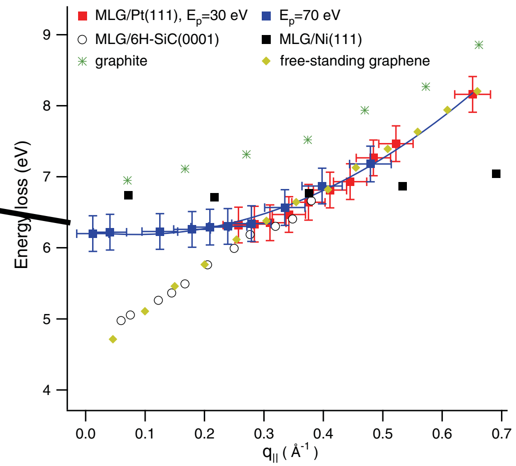
S. Yuan, RR & M. I. Katsnelson. Phys. Rev. B **84**, 035439 (2011)

High energy π-Plasmons

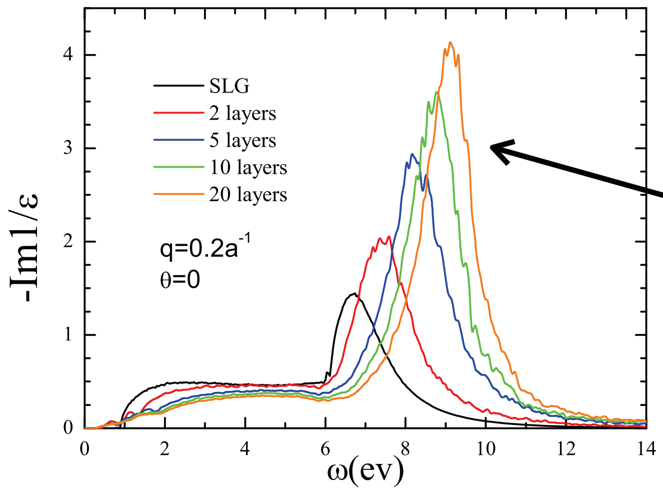
Theoretical RPA results

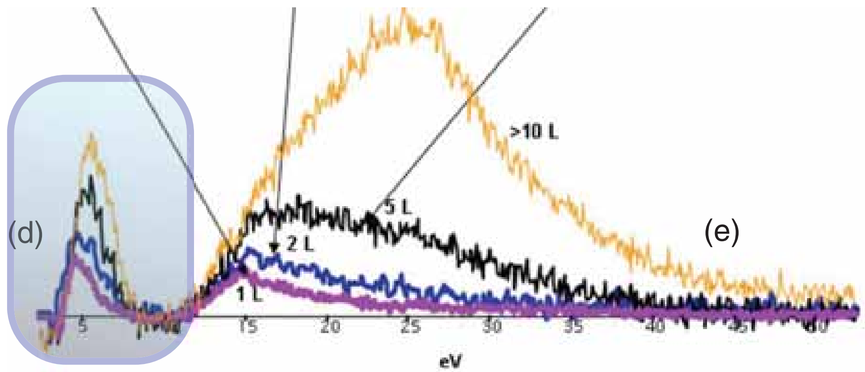


EELS experiments



A. Politano *et al.*, *Plasmonics* (2011)

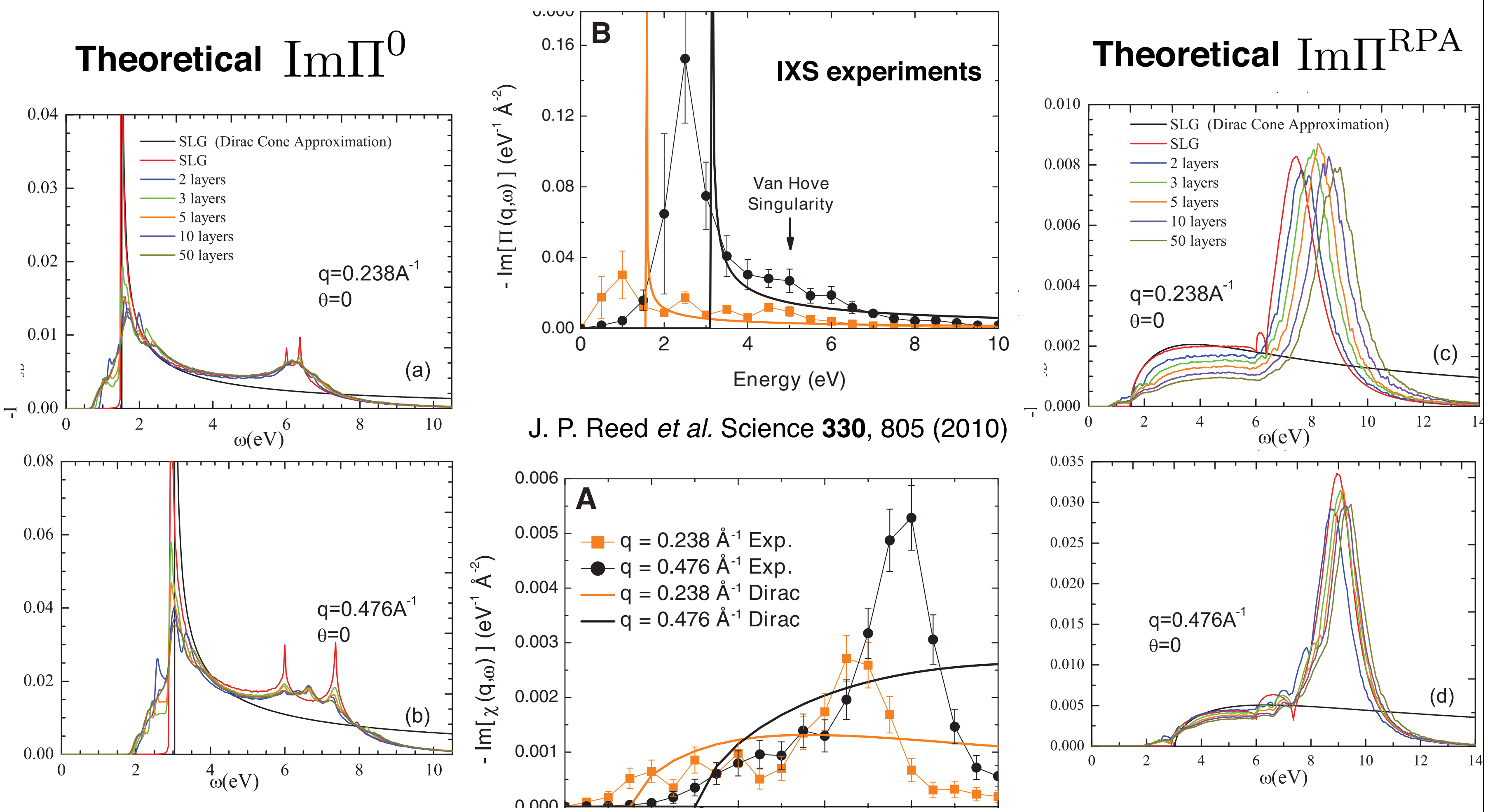




T. Eberlein *et al.*, *Phys. Rev. B* (2008)

S. Yuan, RR & M. I. Katsnelson. Phys. Rev. B **84**, 035439 (2011)

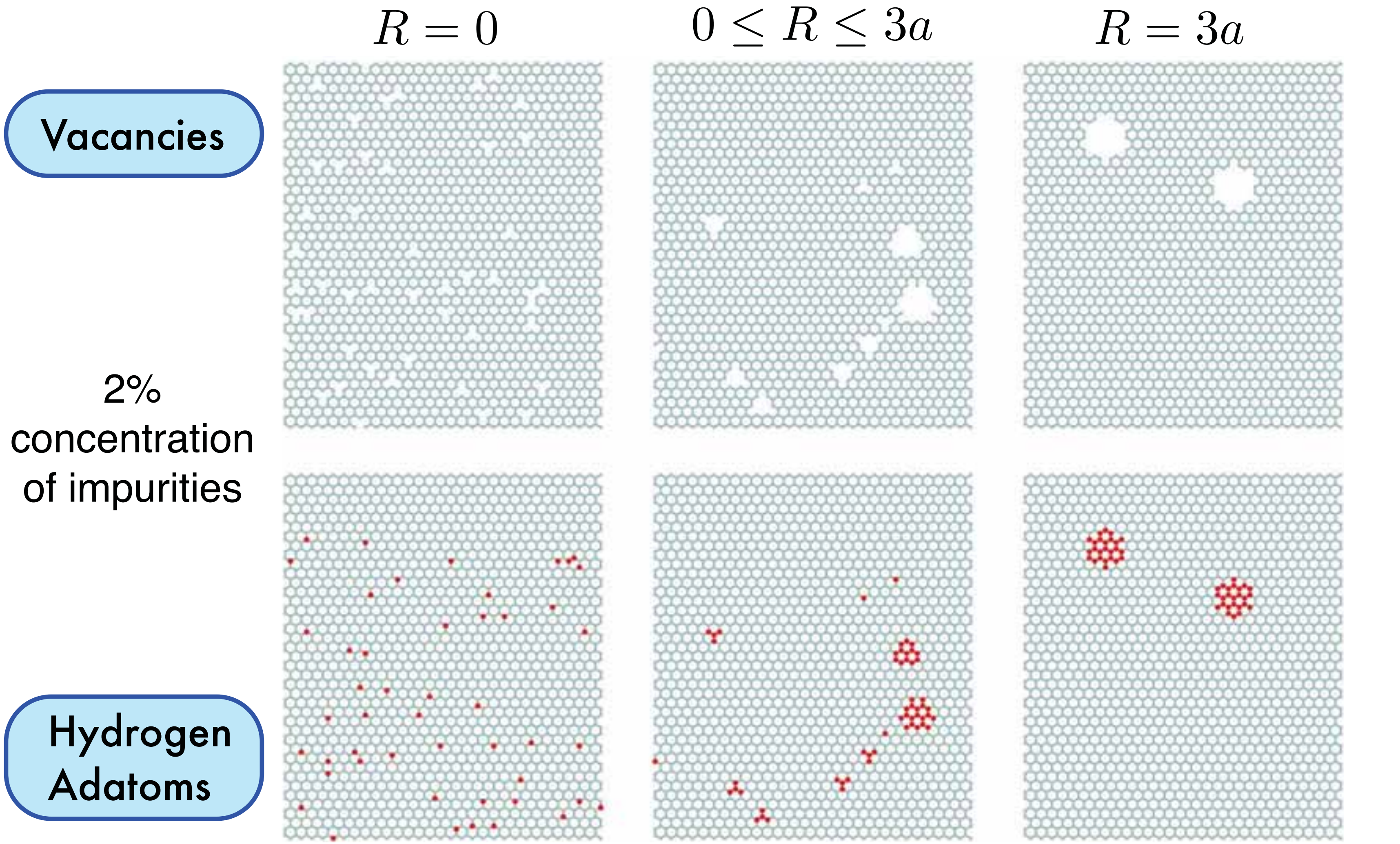
PHES of undoped graphene beyond the Dirac cone approximation



J. P. Reed *et al.* Science **330**, 805 (2010)

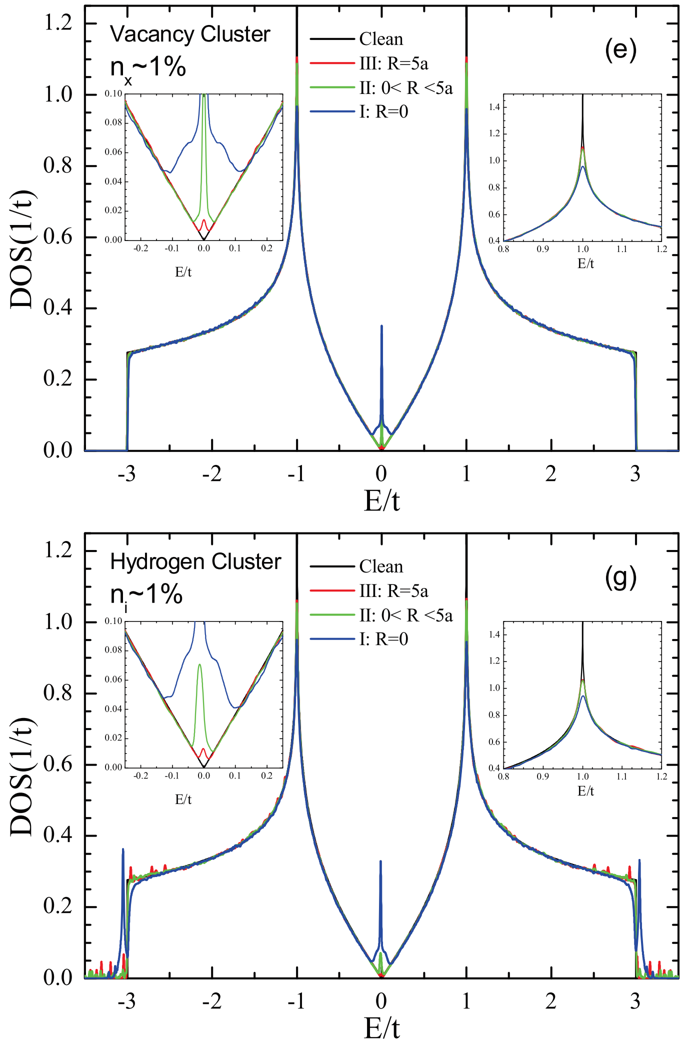
S. Yuan, RR & M. I. Katsnelson. Phys. Rev. B **84**, 035439 (2011)

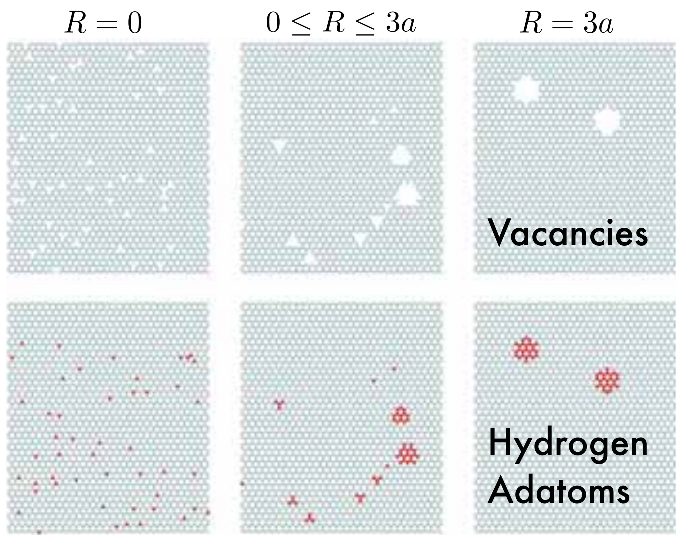
Effect of disordered graphene: the case of resonant impurities

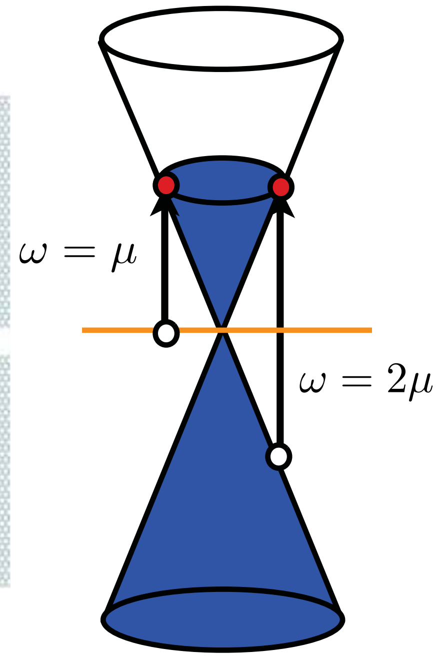


S. Yuan, RR, H. De Raedt & M. I. Katsnelson. Phys. Rev. B **84**, 195418 (2011)

Effect of disordered in graphene: DOS with resonant impurites





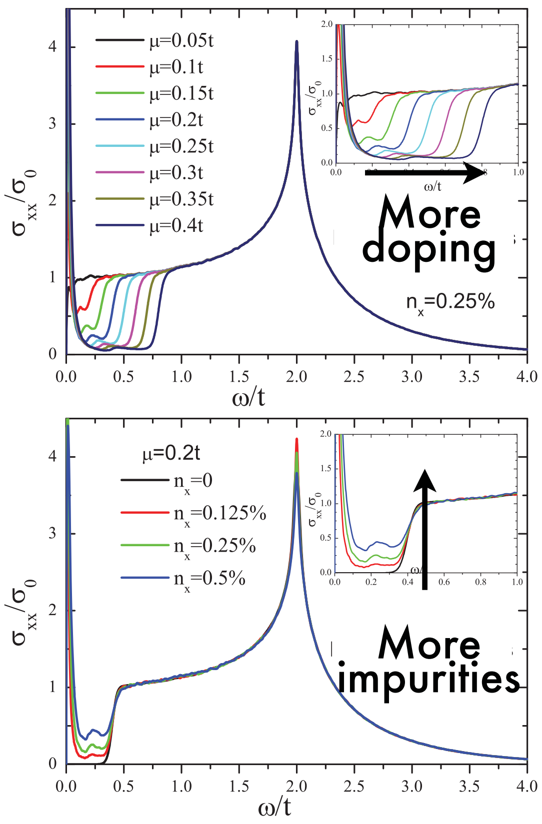


Creation of a zero-energy impurity band due to resonant scatterers

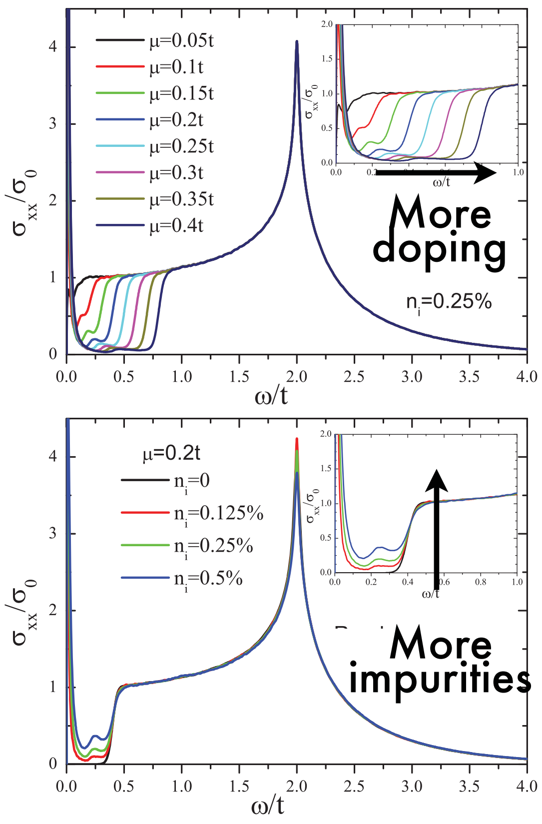
S. Yuan, RR, H. De Raedt & M. I. Katsnelson. Phys. Rev. B **84**, 195418 (2011)

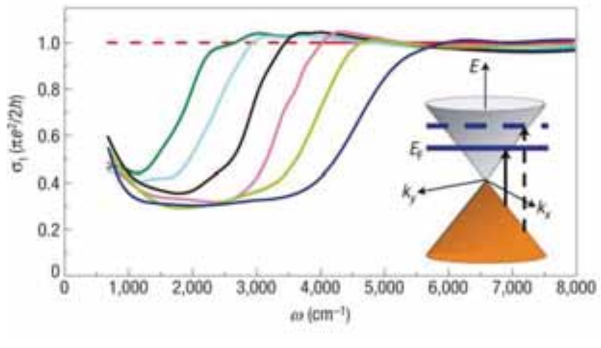
Optical conductivity of disordered graphene: the effect of resonant impurities

Vacancies

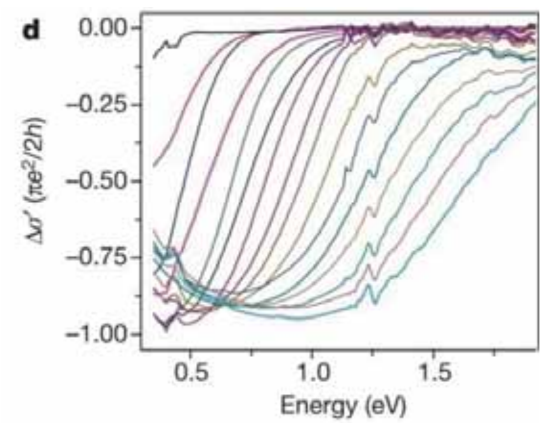


H Adatoms





Z. Q. Li *et al.* Nature Phys. (2008)



Chen *et al.* Nature (2011)

S. Yuan, RR, H. De Raedt & M. I. Katsnelson. Phys. Rev. B **84**, 195418 (2011)

Summary of the PHES with B=0

• High energy resonances (π-plasmons) associated to transitions between Van Hove singularities in undoped graphene. (No phase space for *classical* plasmon within the RPA).

• Theoretical results for the loss function compare well with EELS measurements.

• Effect of resonant scatterers (vacancies, adatoms, etc.) which leads to the creation of zero-energy impurity bands: Background contribution in the optical conductivity.

• Effect of disorder in antidot graphene lattices: stability of the gap

	2DEG	(Doped) graphene
$B = 0, r_s = 0$	Continuum PHES	(Intra-) & inter-band continuum PHES
$B = 0, r_s \neq 0$	Continuum PHES + Plasmon	(Intra-) & inter-band continuum PHES + Plasmons
$B \neq 0, r_s = 0$	Discretized PHES (hints magneto-excitons)	Discretized PHES (hints linear magneto-plasmons)
$B \neq 0, r_s \neq 0$	Discretized PHES (ME) + upper hybrid mode	Discretized PHES (LMP) + upper hybrid mode

S. Yuan, **RR** & M. I. Katsnelson *Phys. Rev. B* **84**, 035439 (2011)
S. Yuan, **RR**, H. De Raedt & M. I. Katsnelson *Phys. Rev. B* **84**, 195418 (2011)
D. Makogon, R. van Gelderen, **RR** & C. Morais Smith *Phys. Rev. B* **84**, 125404 (2011)

Graphene in a strong magnetic field

Tight-binding model at B≠0: Peierls subtitution

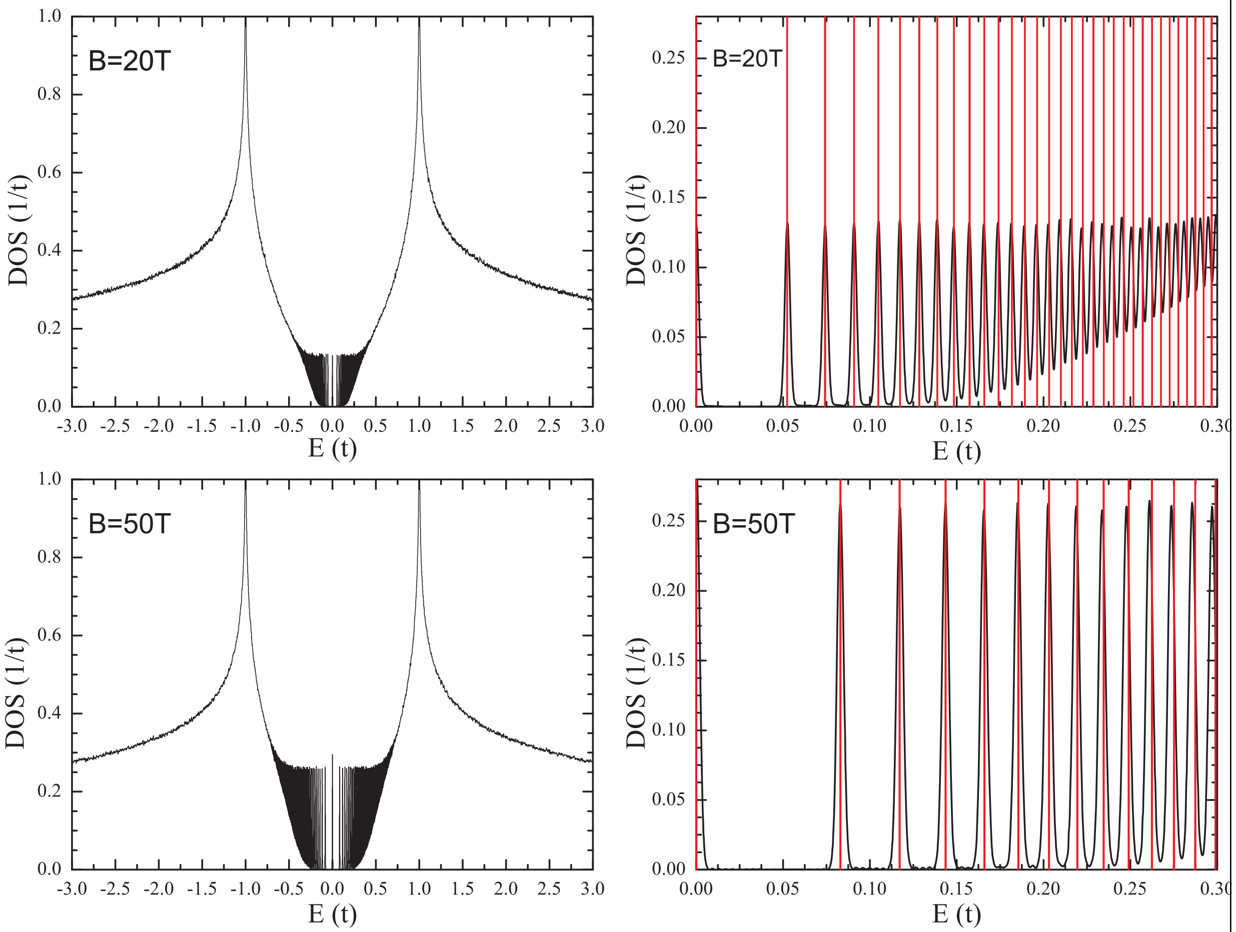
$$t_{ij} \to t_{ij} \exp\left(i \frac{2\pi}{\Phi_0} \int_{\mathbf{R}_i}^{\mathbf{R}_j} \mathbf{A} \cdot dl \right)$$

- Good agreement with the Dirac-cone approximation for the low energy Landau levels (LL)

$$\epsilon_{\lambda,n} = \lambda \epsilon_n = \lambda \frac{v_F}{l_B} \sqrt{2n} \propto \sqrt{Bn}.$$

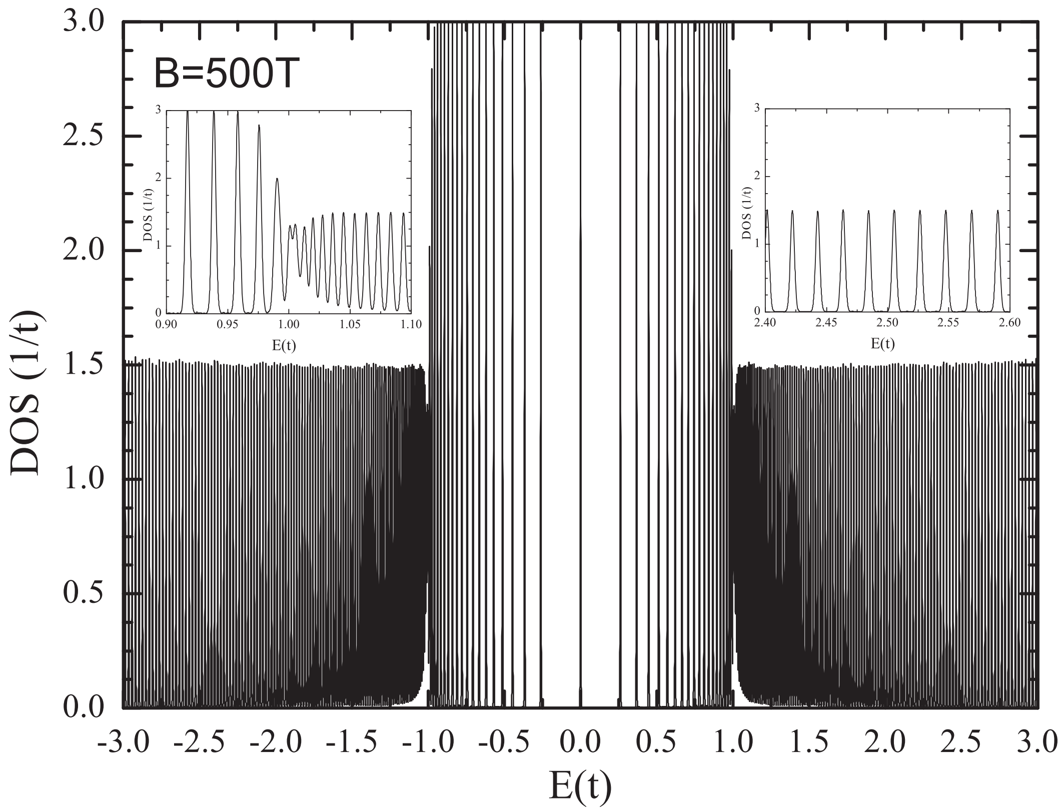
up to $\epsilon \sim 0.3t \sim 1\text{eV}$

see experimental results by P. Plochocka *et al.*, *Phys. Rev. Lett.* **100**, 087401 (2008)



S. Yuan, RR & M. I. Katsnelson, *Solid State Comm.* **152**, 1446 (2012)

DOS in the ultra-high magnetic field limit



- **Experimental realization in "artificial graphene"**

$$a \sim 130 \text{ nm}$$
$$a_0 \sim 0.14 \text{ nm}$$

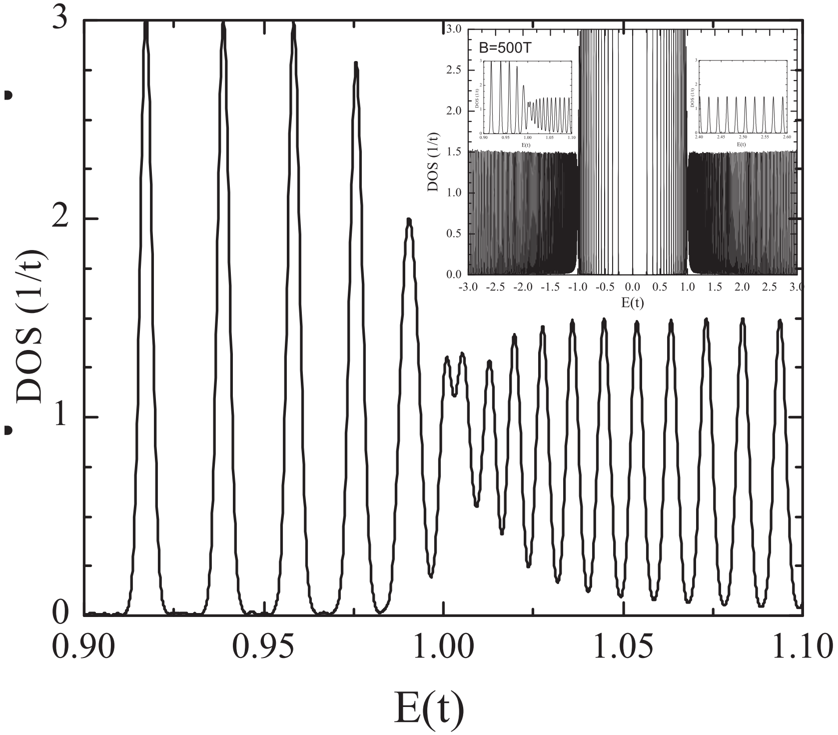
Effective fields $(a/a_0)^2 \sim 8 \times 10^5$ larger than in *real* graphene

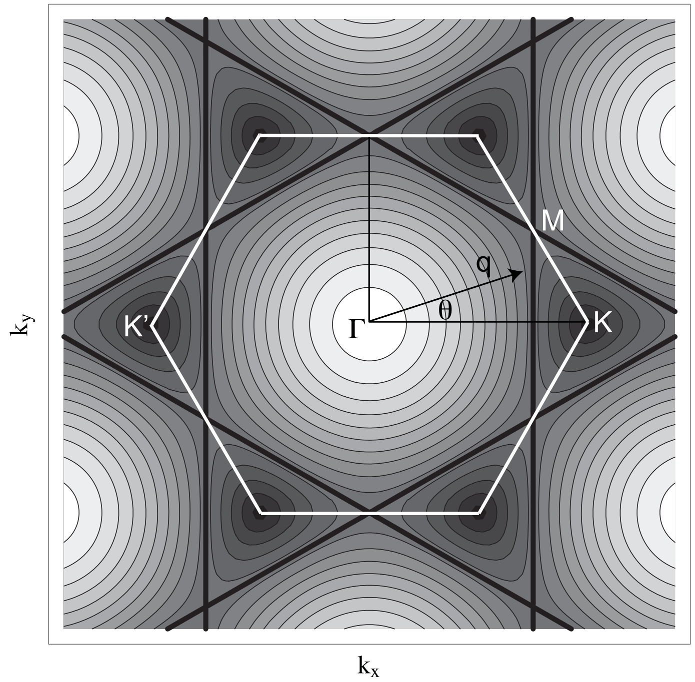
A. Singha *et al.* Science (2011)

- Non-trivial Landau level quantization of the spectrum around the Van Hove singularity with two different sets of LLs that merge at the saddle point

S. Yuan, RR & M. I. Katsnelson, *Solid State Comm.* **152**, 1446 (2012)

Semiclassical Landau level quantization





- SC condition for cyclotron orbit

$$S(C) = \frac{2\pi}{l_B^2}\left(n + \frac{1}{2} - \frac{\Gamma(C)}{2\pi}\right)$$

$$S(C) = \iint_{\epsilon(k_x,k_y)\leq\,\epsilon_n} dk_x dk_y$$

- Berry phase

$$\Gamma \rightarrow \Gamma(C) = 0$$

$$K, K' \rightarrow \Gamma(C) = \pm\pi$$

- SC energy of the Landau levels

$$\epsilon_n = S^{-1}\left(\frac{2\pi}{l_B^2}\left[n + \frac{1}{2} - \frac{\Gamma(C)}{2\pi}\right]\right)$$

S. Yuan, RR & M. I. Katsnelson, *Solid State Comm.* **152**, 1446 (2012)

PHES of graphene in a magnetic field using a full π-band tight-binding model

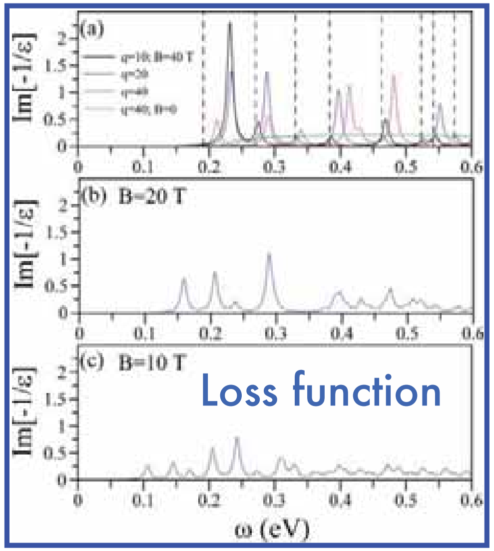
- This problem has been first considered in graphene here...

Plasma Excitations in Graphene: Their Spectral Intensity and Temperature Dependence in Magnetic Field

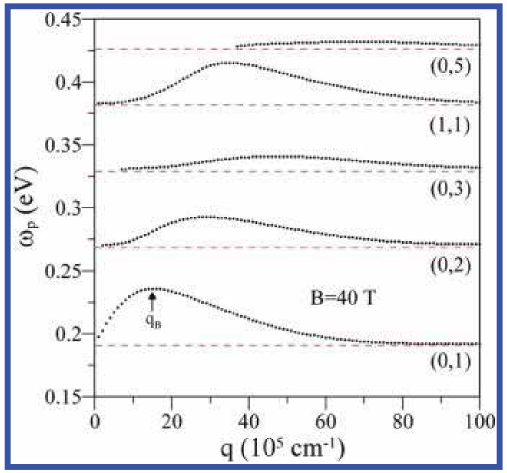
Jhao-Ying Wu,† Szu-Chao Chen,† Oleksiy Roslyak,‡ Godfrey Gumbs,*,‡ and Ming-Fa Lin*,†

†Department of Physics, National Cheng Kung University, Tainan, Taiwan 701, and ‡Department of Physics and Astronomy, Hunter College at the City University of New York, 695 Park Avenue, New York, New York 10065, United States

VOL. 5 ▪ NO. 2 ▪ 1026–1032 ▪ 2011 ACSNANO

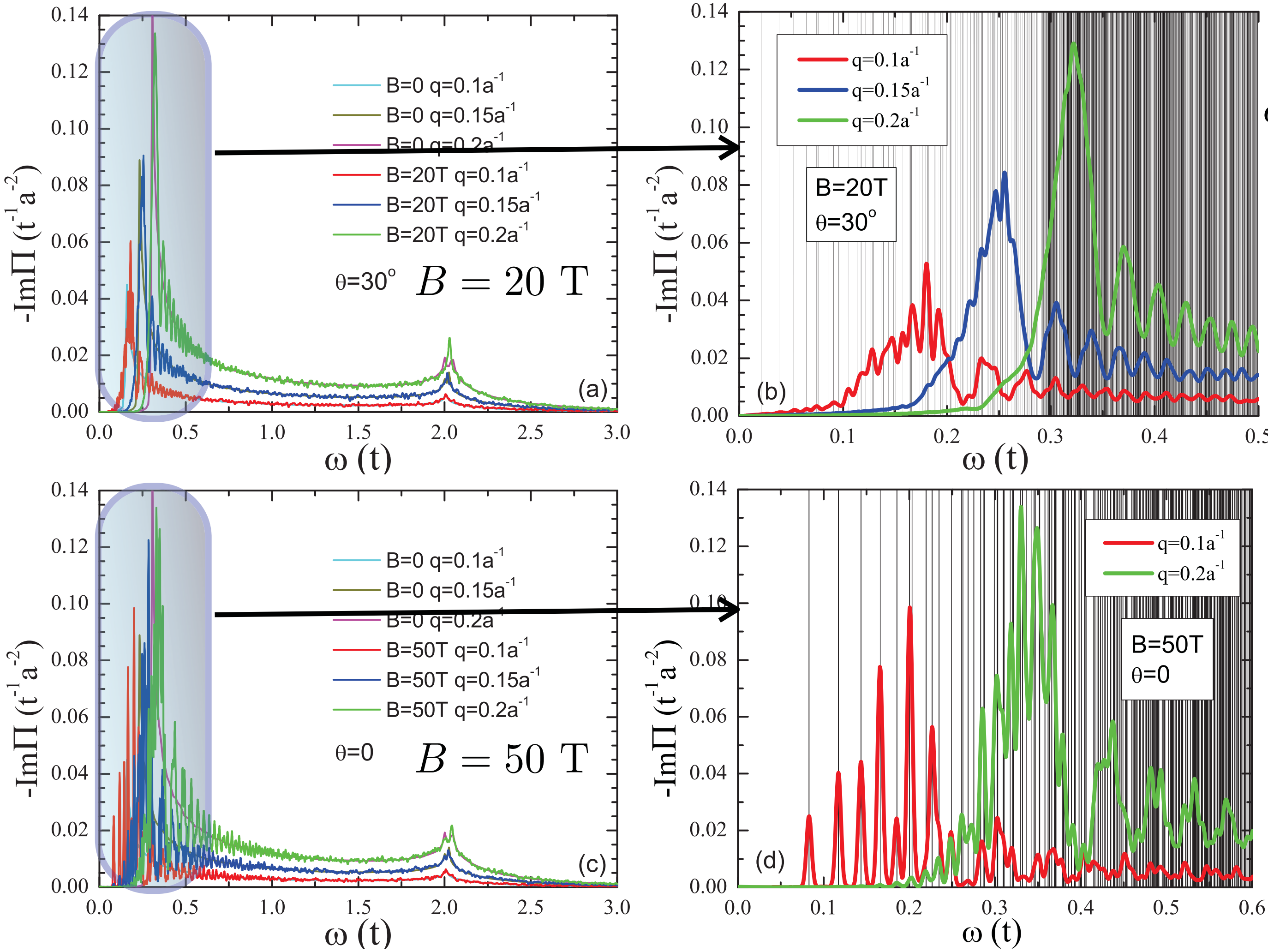


Loss function

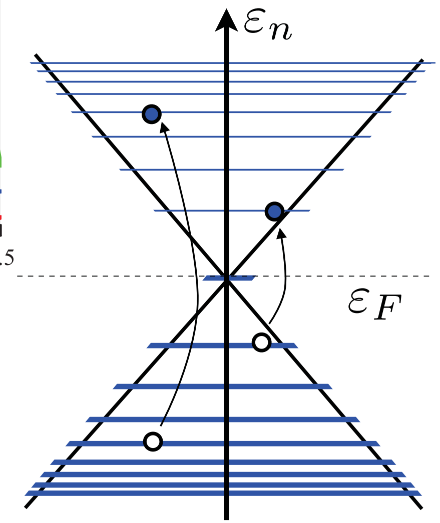


Dispersion relation of magneto-plasmons

PHES of graphene in a magnetic field using a full π-band tight-binding model



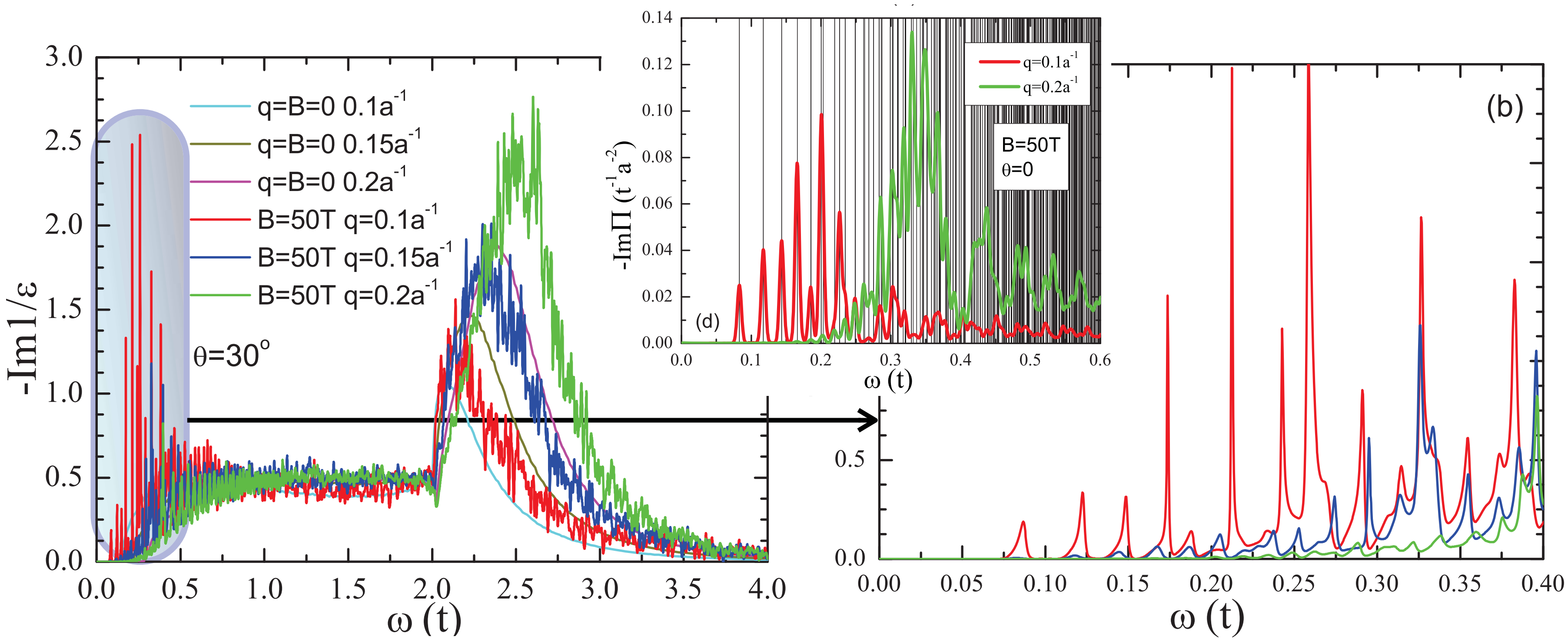
$$\omega_{n,n'} = \sqrt{2}(v_F/l_B)(\sqrt{n'}+\sqrt{n}).$$



S. Yuan, RR & M. I. Katsnelson, *Solid State Comm.* **152**, 1446 (2012)

Loss function of graphene in a magnetic field using a full π-band tight-binding model

el-el interactions within the RPA

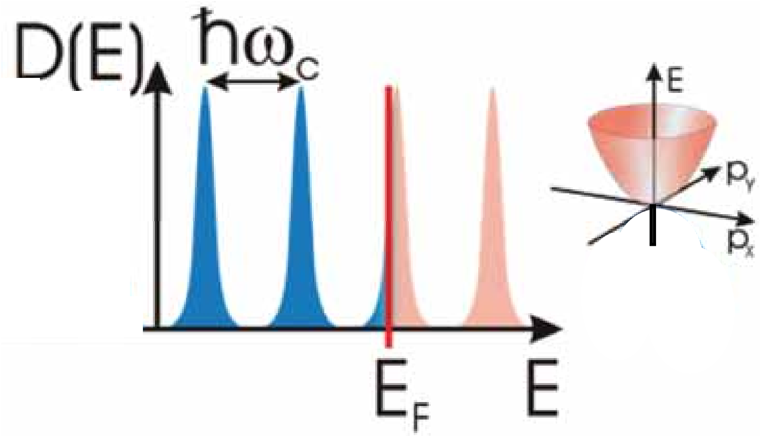


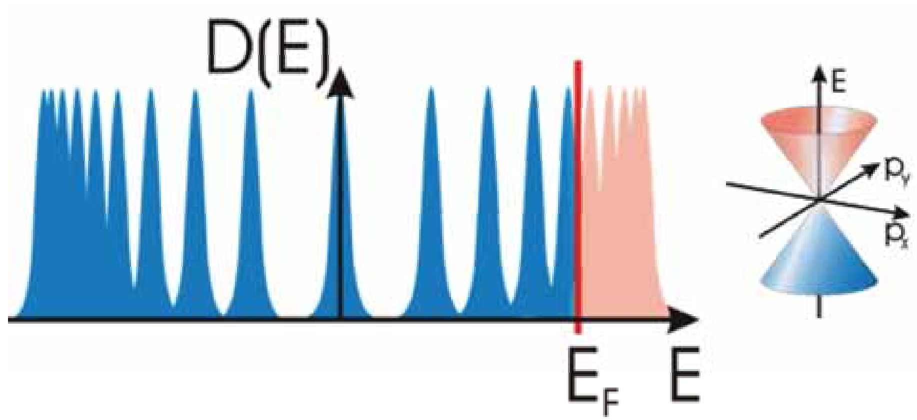
The number of resonant peaks does not coincides with the number of all possible inter-LL transitions

S. Yuan, RR & M. I. Katsnelson, *Solid State Comm.* **152**, 1446 (2012)

Graphene's "relativistic" Landau levels: B≠0

Standard 2DEG B≠0 **Doped graphene**





$$\varepsilon_n = \left(n + \frac{1}{2}\right)\frac{\hbar e B}{m_b}$$
$$n = 0, 1, 2, \ldots$$
$$\hbar\omega_c = \hbar\frac{eB}{m_b}$$
$$m_b = \text{constant}$$

$$\varepsilon_{n,\pm} = \pm v_F\sqrt{2\hbar e B n}$$
$$n = 0, 1, 2, \ldots$$
$$\hbar\omega_c(\varepsilon_F) = \hbar\frac{eBv_F^2}{\varepsilon_F} \neq v_F\sqrt{2\hbar e B}$$
$$v_F = \text{constant} \approx 10^6 m/s$$

K. S. Novoselov *et al.* *Nature* **438**, 197 (2005); Zhang *et al.* *Nature* **438**, 201 (2005)

PHES of a standard 2DEG in a magnetic field

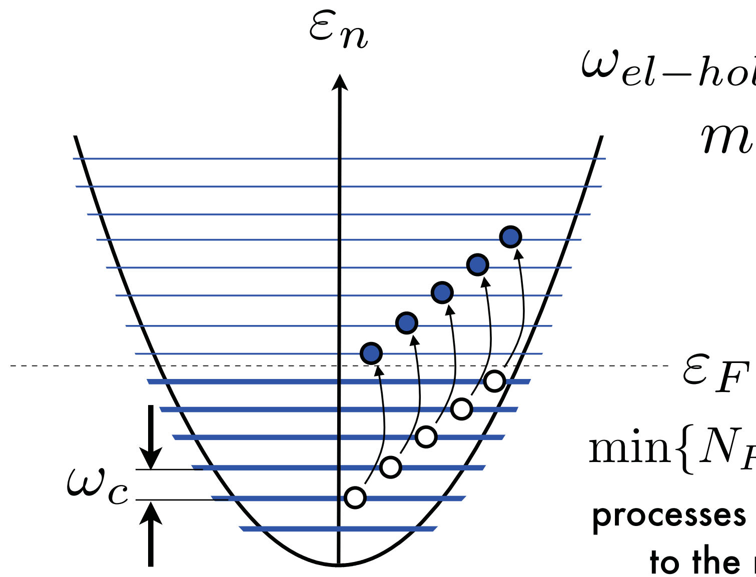
Landau levels (LLs)

$$\varepsilon_n = \hbar \frac{eB}{m_b}\left(n + \frac{1}{2}\right)$$

$$B \neq 0$$

"Density independent" cyclotron frequency

$$\omega_c = \frac{eB}{m_b}$$

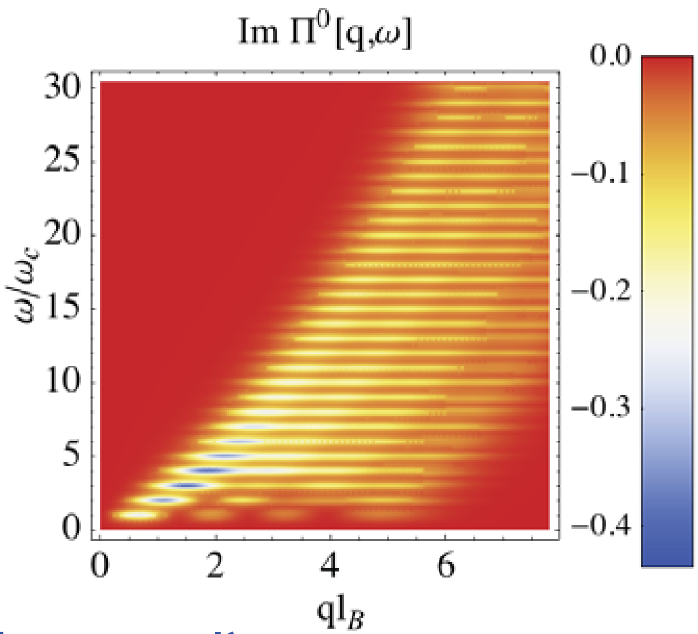


$$\omega_{el-hol} \sim m\omega_c$$

$$m \in \mathbb{N}$$

$$\min\{N_F + 1, m\}$$

processes contributing to the m'th line



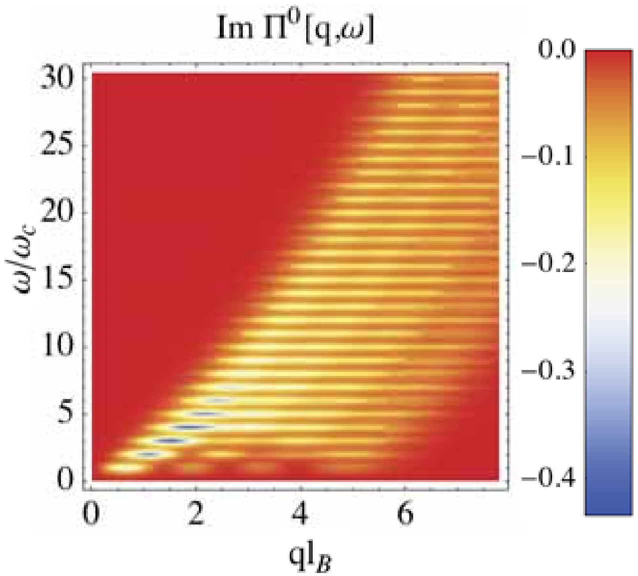
LL quantization

+

Equidistant LL separation

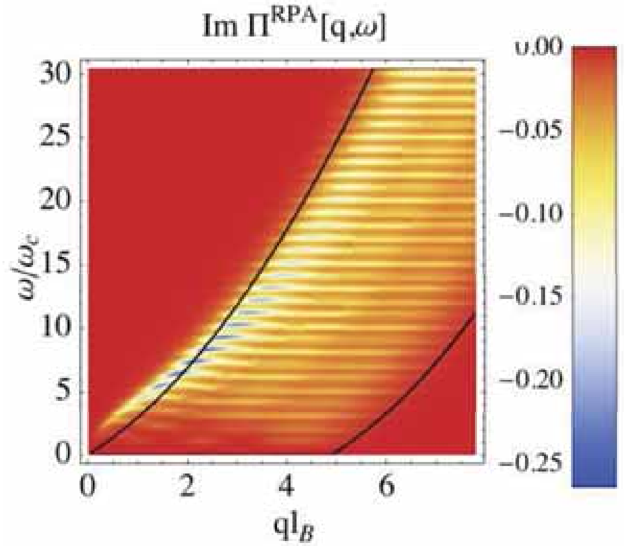
(horizontal) magneto-excitons

Interacting 2DEG in a magnetic field: RPA theory



$$r_s \equiv \frac{2 m_b e^2}{\hbar^2 \varepsilon_b k_F} \approx 3$$

el-el interactions



- Interactions lead to a transfer of spectral weight from the long-wave length region of the PHES to the plasmon mode, modified by the magnetic field: known as upper hybrid (UH) mode in plasma phys.
- The magneto-excitons acquire a dispersion due to the inclusion of Coulomb interaction.

UH mode

$$\omega_{UH} = \sqrt{\omega_c^2 + \omega_{p,cl}^2}$$

$$\omega_c^2 = \left(\frac{eB}{m_b}\right)^2$$

$$\omega_{p,cl}^2 \simeq \frac{2\pi e^2 n_0}{\varepsilon_b m_b} q$$

Particle-hole polarization in graphene: B≠0

Decomposition in inter- and intra-band contributions

$$\Pi_n^{\lambda}(\mathbf{q},\omega) = \sum_{\lambda'}\sum_{n'=0}^{n-1}\Pi_{nn'}^{\lambda\lambda'}(\mathbf{q},\omega) + \sum_{\lambda'}\sum_{n'=n+1}^{N_c}\Pi_{nn'}^{\lambda\lambda'}(\mathbf{q},\omega) + \Pi_{nn}^{\lambda-\lambda}(\mathbf{q},\omega)$$

$$\Pi^0(\mathbf{q},\omega) = \sum_{n=1}^{N_F}\Pi_n^{\lambda_F}(\mathbf{q},\omega) + \Pi^{vac}(\mathbf{q},\omega)$$

Vacuum (inter-band) contribution

$$\Pi^{vac}(\mathbf{q},\omega) \equiv -\sum_{n=1}^{N_c}\Pi_n^{\lambda=1}(\mathbf{q},\omega)$$

Filling factor

$$\nu = 4N_F + 2$$

Cutoff in the LL index

$$N_c \sim \frac{10^4}{B[T]}$$

$$\Pi_{nn'}^{\lambda\lambda'}(\mathbf{q},\omega) \equiv \frac{\mathcal{F}_{nn'}^{\lambda\lambda'}(\mathbf{q})}{\lambda\xi_n - \lambda'\xi_{n'} + \omega + i\delta\mathrm{sgn}(\omega)}$$

Form factor of the polarization function at B ≠ 0

$$\mathcal{F}_{nn'}^{\lambda\lambda'}(\mathbf{q}) = \frac{e^{-l_B^2q^2/2}}{2\pi l_B^2}\left(\frac{l_B^2q^2}{2}\right)^{n_>-n_<}\left\{\lambda 1_n^* 1_{n'}^*\sqrt{\frac{(n_<-1)!}{(n_>-1)!}}\left[L_{n_<-1}^{n_>-n_<}\left(\frac{l_B^2q^2}{2}\right)\right] + \lambda' 2_n^* 2_{n'}^*\sqrt{\frac{n_<!}{n_>!}}\left[L_{n_<}^{n_>-n_<}\left(\frac{l_B^2q^2}{2}\right)\right]\right\}^2$$

PHES of graphene in a magnetic field

Landau levels (LLs)

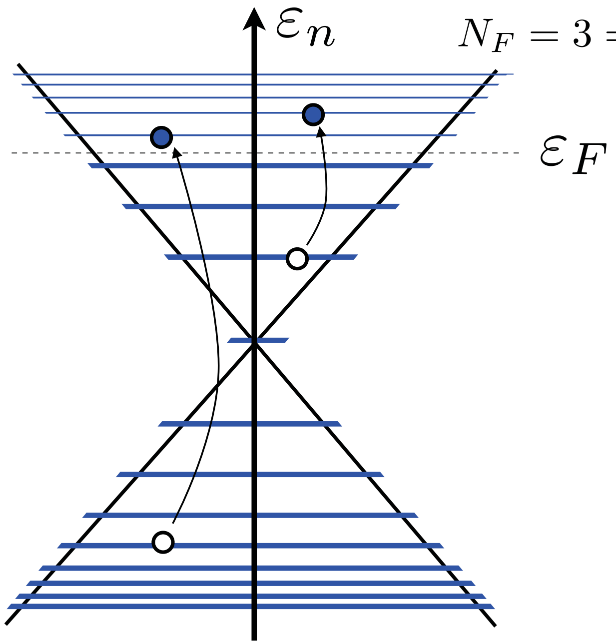
$$\varepsilon_{\lambda,n} = \lambda\hbar\frac{v_F}{l_B}\sqrt{2n}$$

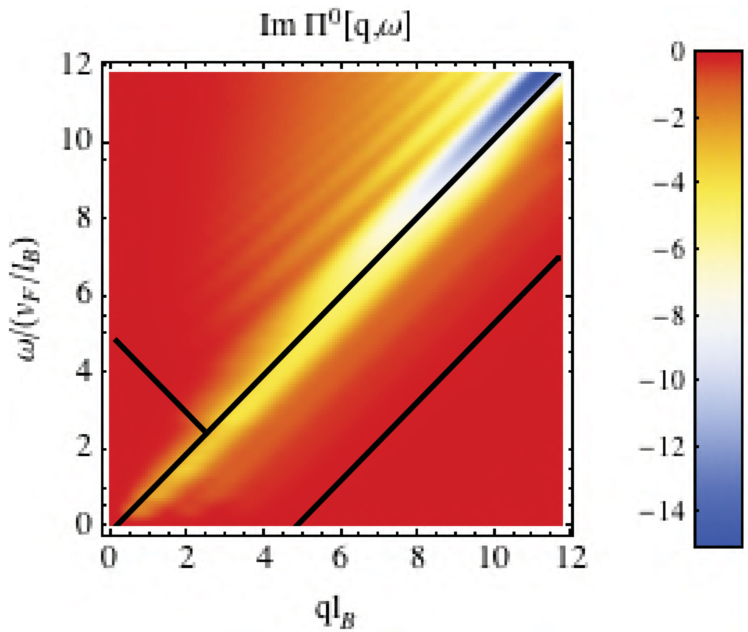
$$B \neq 0$$

"Density dependent" cyclotron frequency

$$\omega_c(\varepsilon_F) = \frac{eB}{\varepsilon_F/v_F^2}$$

$N_F = 3 \Rightarrow 2k_F = 2\sqrt{7} \simeq 5.3 l_B$

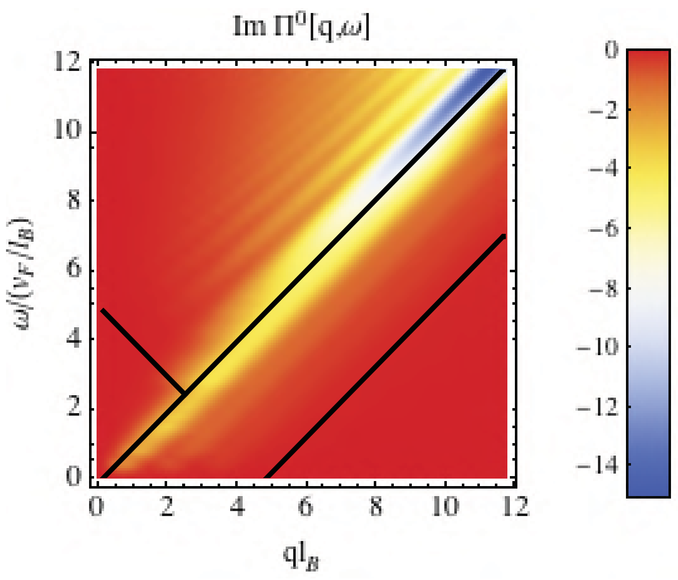




LL quantization
+
Non-equidistant LL separation
+
Chirality factor

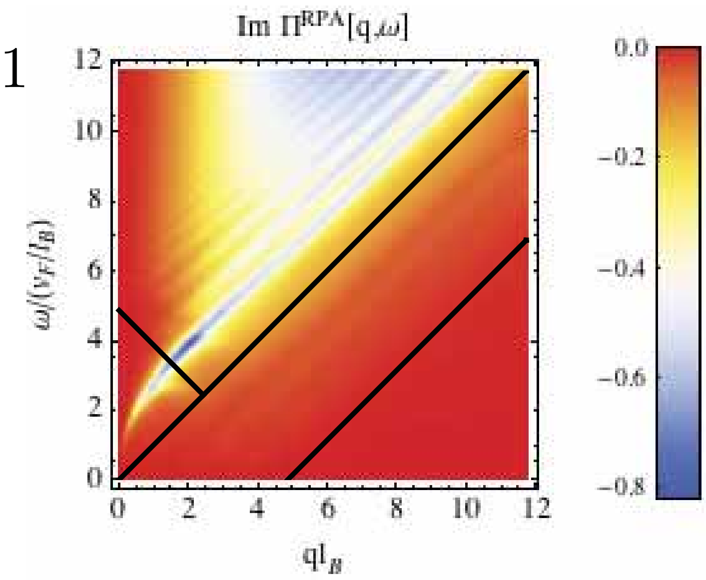
- blurred horizontal magneto-excitons
- precursor of dispersive modes: (linear) magneto-plasmons

Interacting graphene in a magnetic field: RPA theory



$$r_s \equiv \frac{e^2}{\epsilon_b \hbar v_F} \approx 1$$

el-el interactions



- Interactions lead to a transfer of spectral weight from the PHES to the upper-hybrid mode
- Relatively weak renormalization of the intra-band region of the spectrum.
- The linear magneto-plasmons are clearly pronounced due to the inclusion of Coulomb interaction.

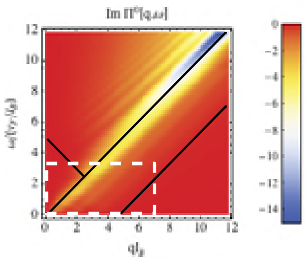
Zoom of the low energy PHES: Graphene vs. 2DEG

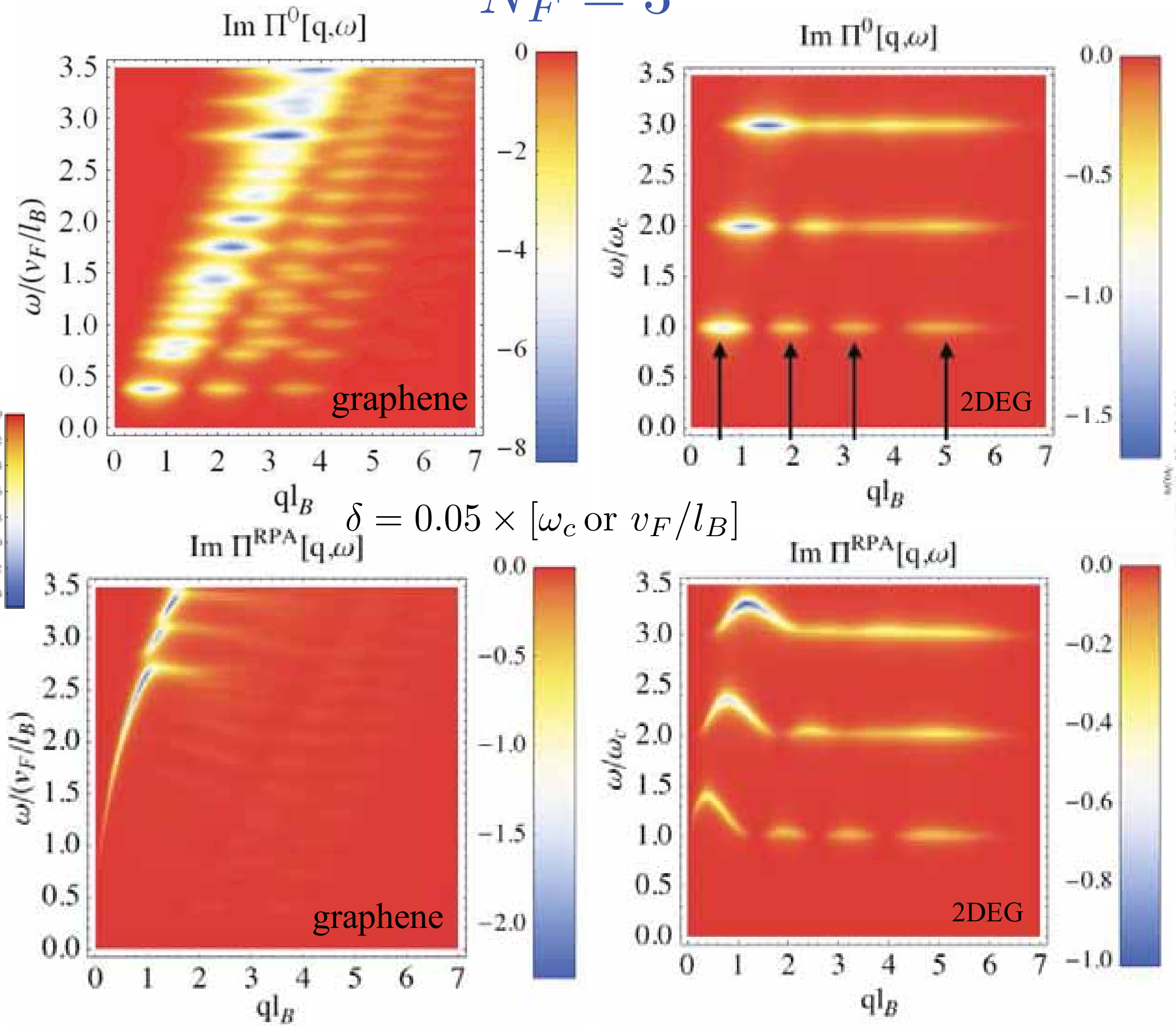
Graphene | 2DEG

$N_F = 3$

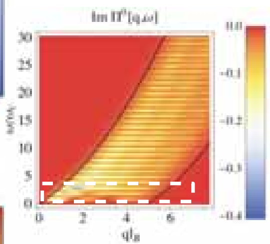
Non-interacting PHES

Interacting PHES

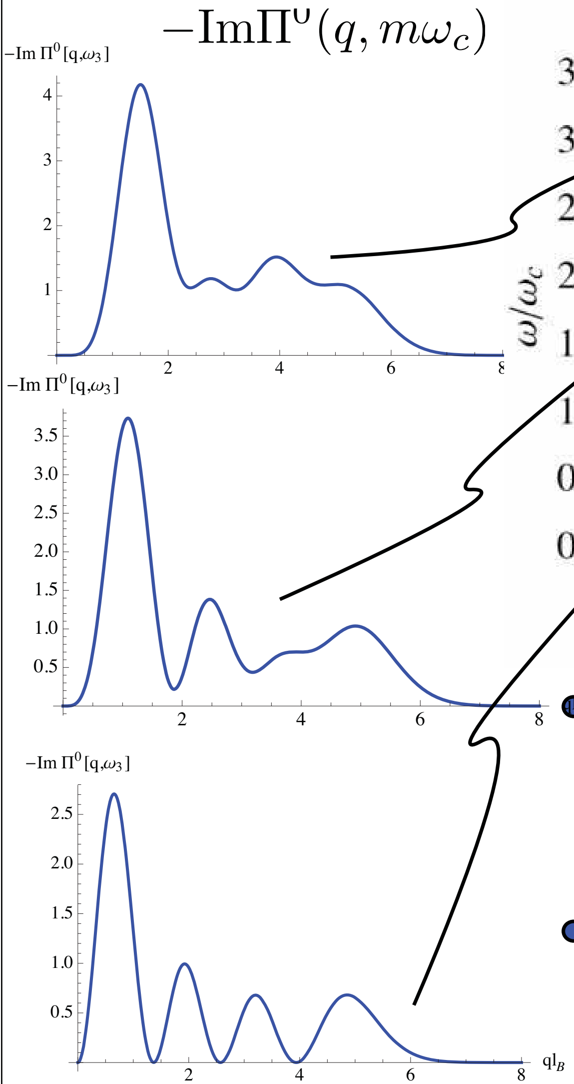


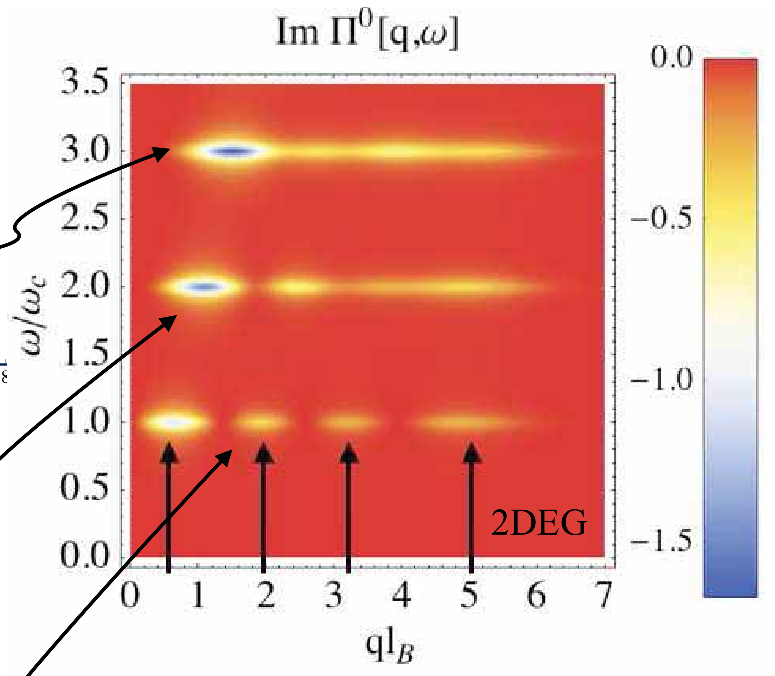


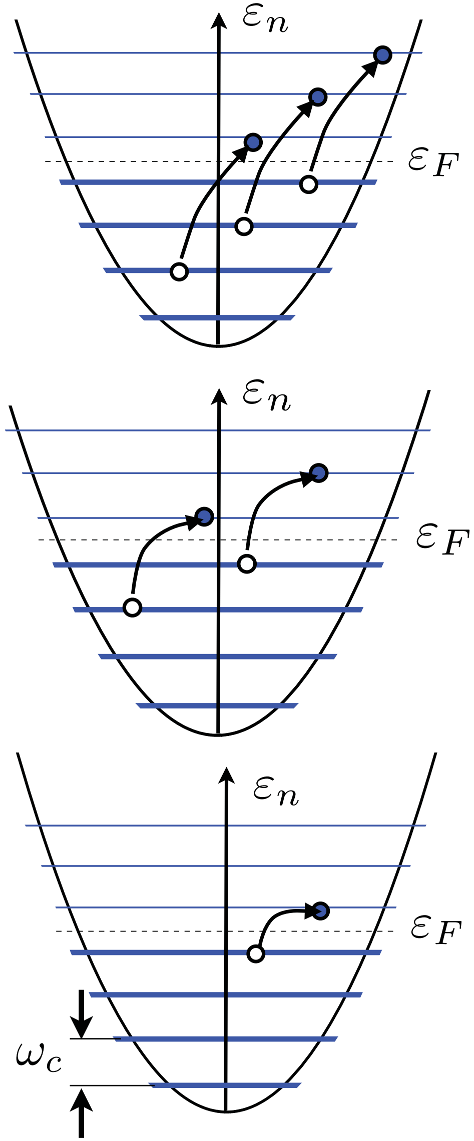
$\delta = 0.05 \times [\omega_c \, \text{or} \, v_F/l_B]$



Structure of the PHES: 2DEG vs. Graphene

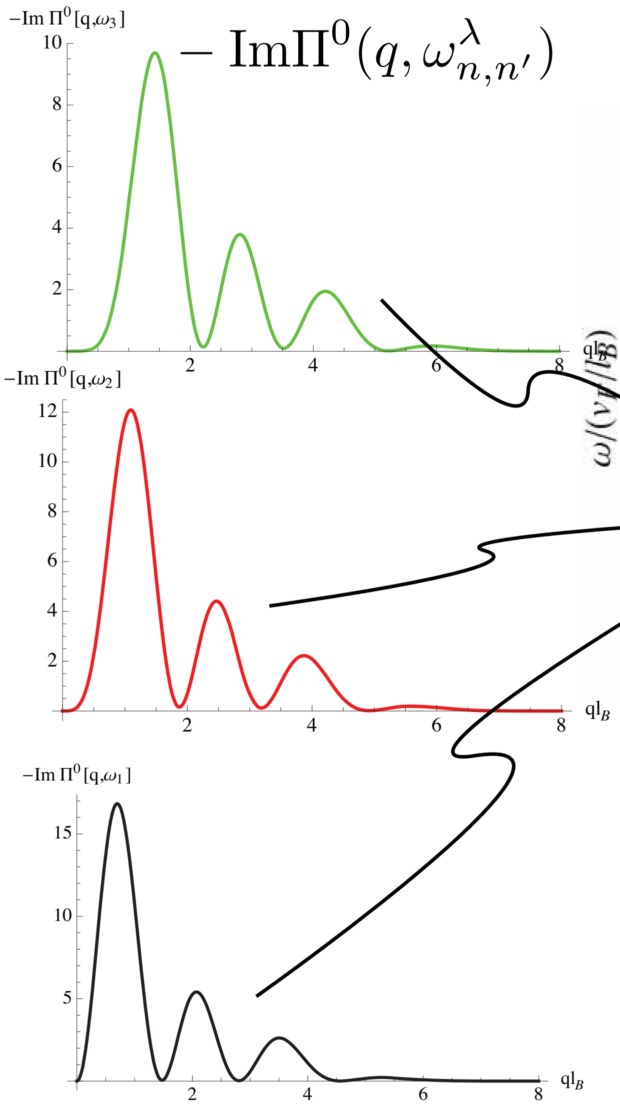


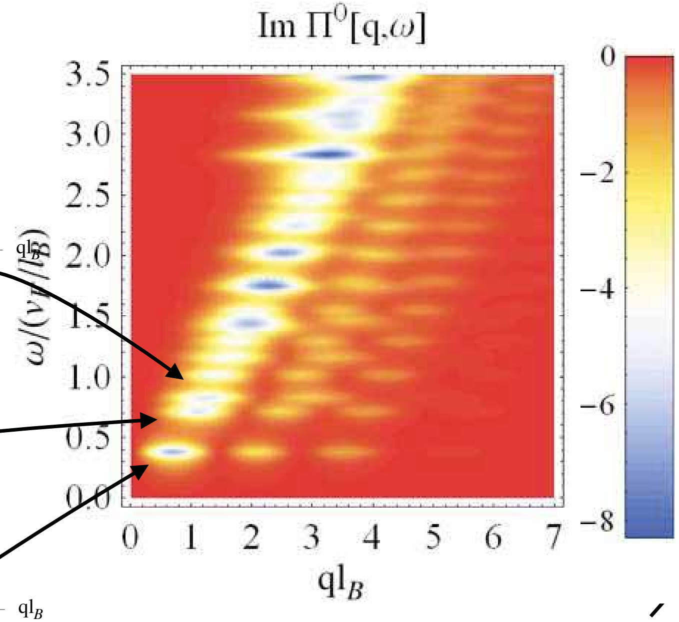


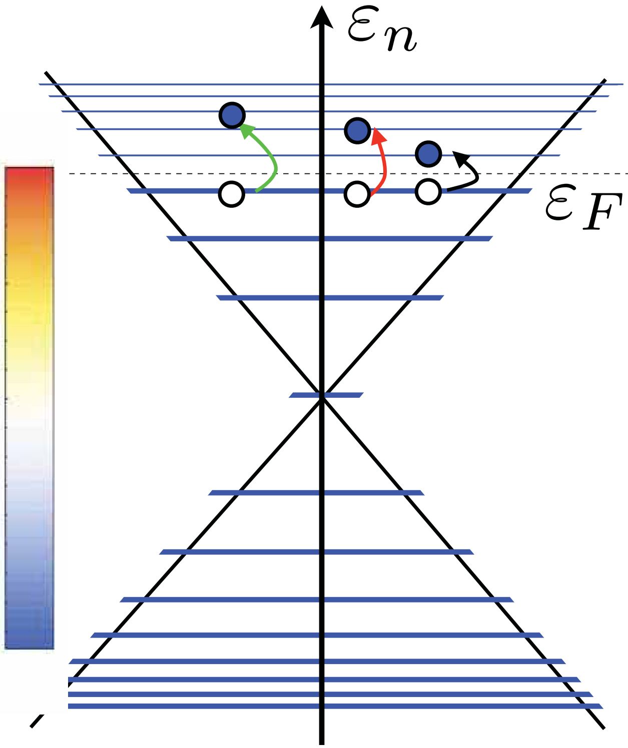


- Nodes of the electronic wave-function in a magnetic field lead to pseudo-quantization of the momentum along a single "magneto-exciton"
- However, effect only appreciable in the 2DEG for the first magneto-exciton due to overlapping of different processes at higher energies.

Structure of the PHES: 2DEG vs. Graphene







- Clear "pseudo-quantization" of the momentum along every single horizontal line associated to a given particle-hole excitation.

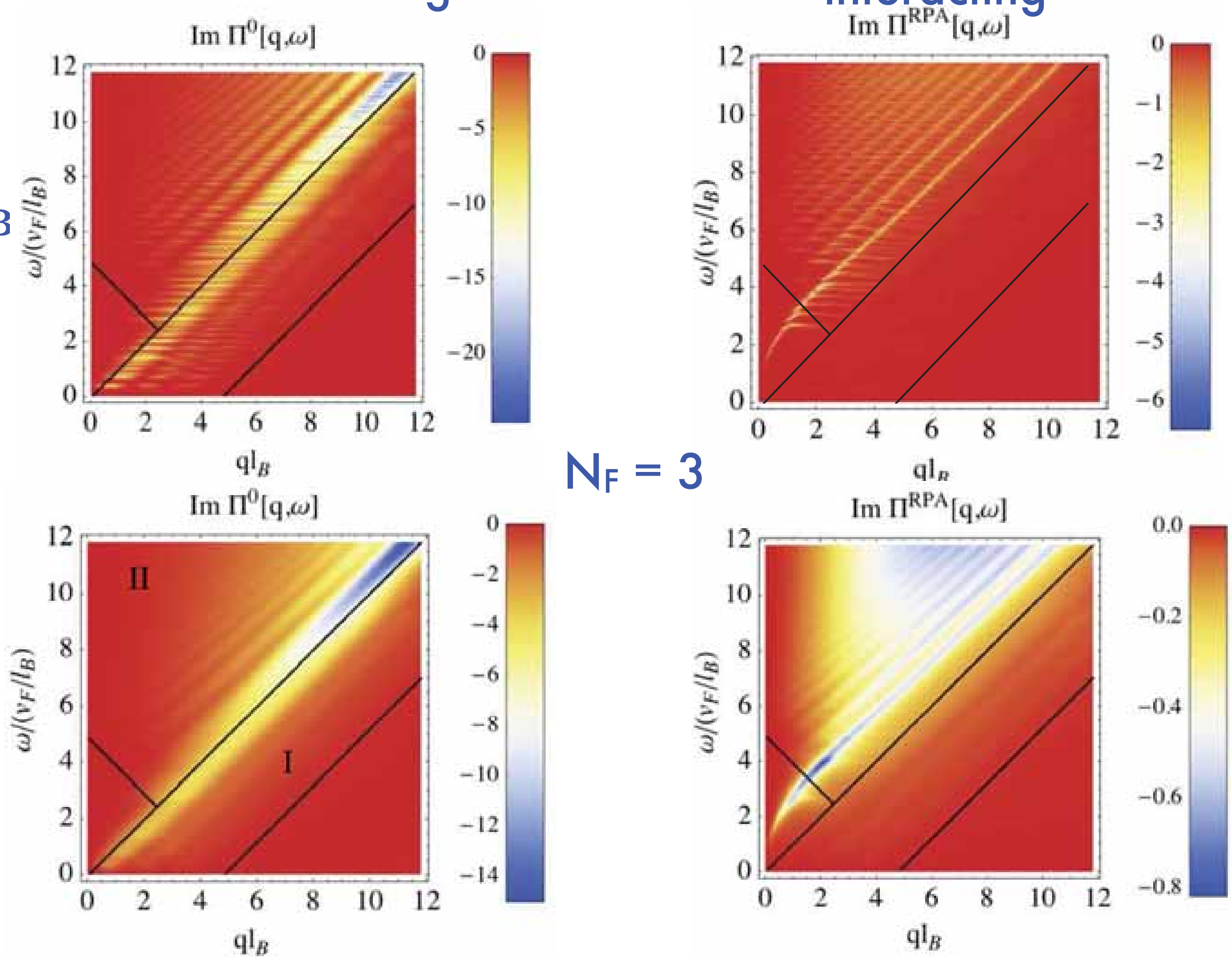
- Stacking of the horizontal magneto-excitons at higher energies due to the large number of processes contributing to the same energy window. Effect of non-equidistant LLs in graphene.

Effect of disorder: Landau level broadening

non-interacting | interacting

$\delta = 0.02 v_F / l_B$

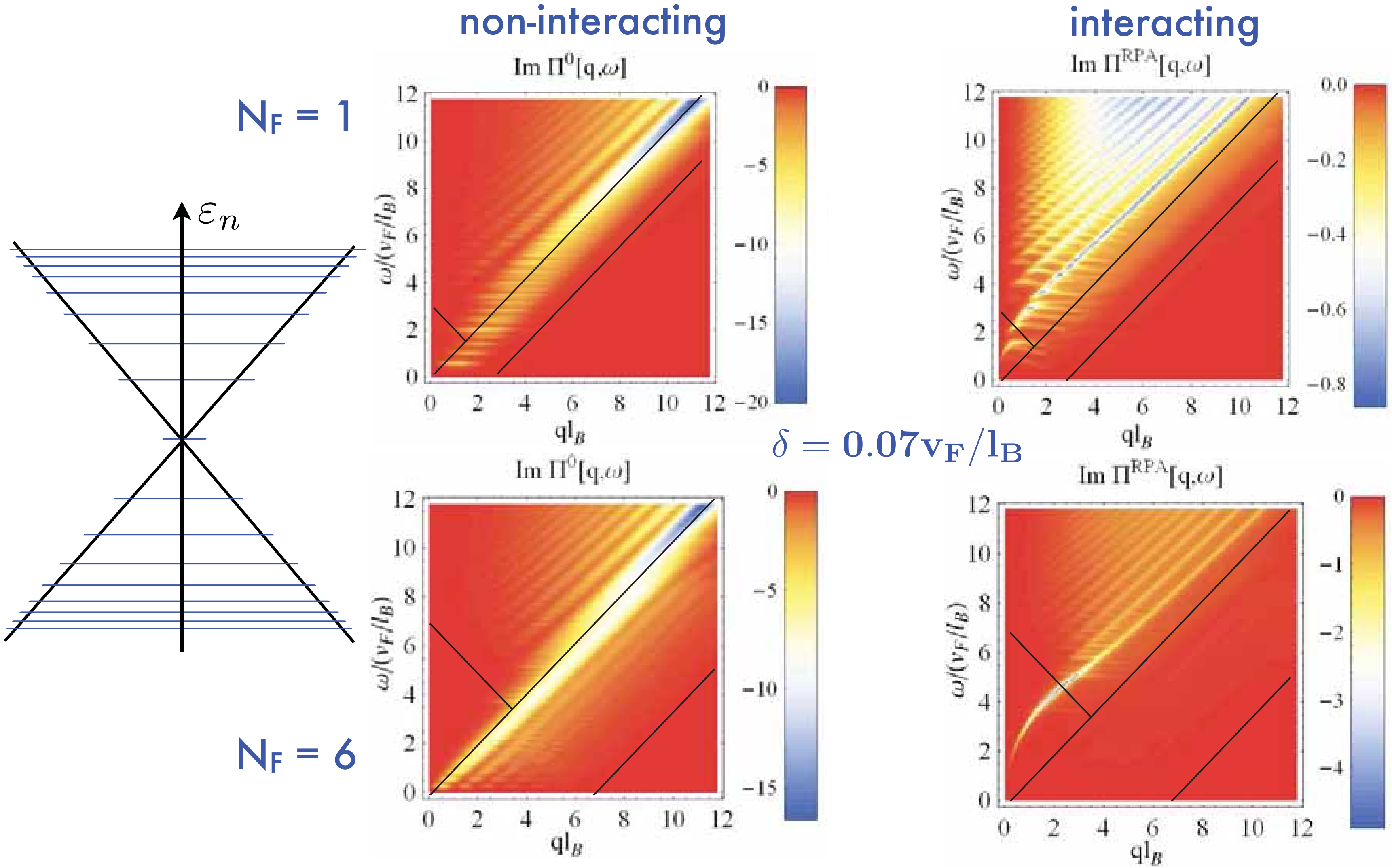
$\delta = 0.2 v_F / l_B$



$N_F = 3$

- Disorder favors linear magneto-plasmons against horizontal magneto-excitons.

Effect of doping: Landau level filling



- Doping favors linear magneto-plasmons against horizontal magneto-excitons.

Hydrodynamics: upper hybrid mode (plasmon)

- Euler and continuity equations:

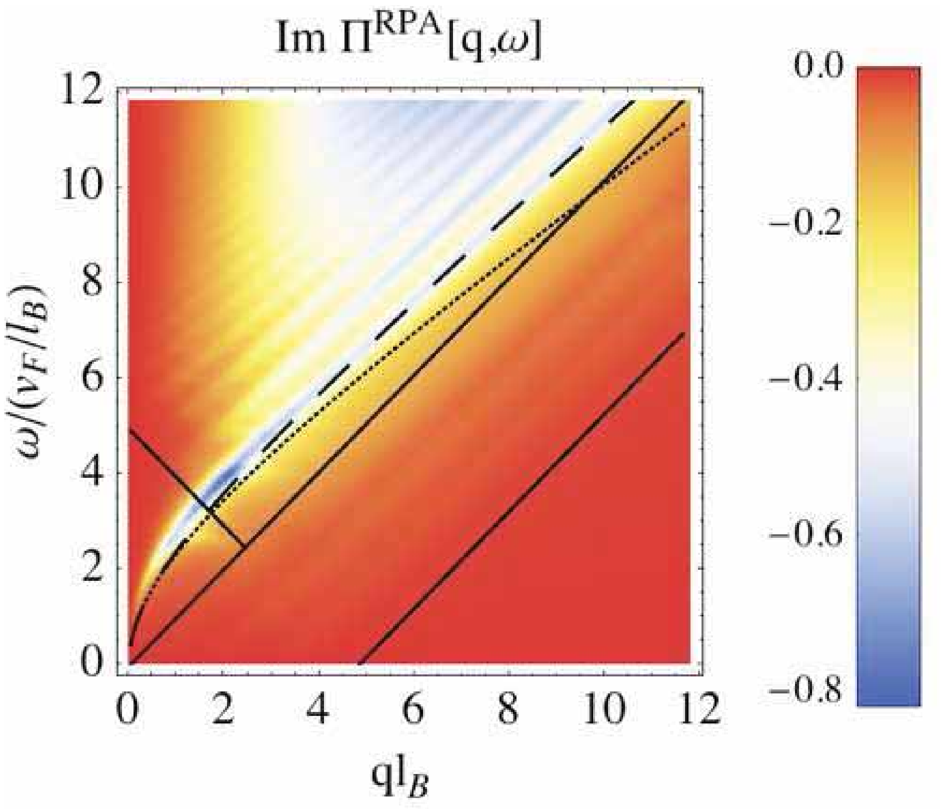
$$\frac{d}{dt}\left(\frac{\varepsilon_F}{v_F^2}\mathbf{J}(\mathbf{r},t)\right) = e\boldsymbol{\nabla}P(\mathbf{r},t) + en(\mathbf{r},t)\boldsymbol{\nabla}\int d\mathbf{r}'\frac{e^2}{|\mathbf{r}-\mathbf{r}'|}[n(\mathbf{r}',t)-n_0] - e\mathbf{J}(\mathbf{r},t)\times\mathbf{B},$$

$$\frac{\partial n(\mathbf{r},t)}{\partial t} = \frac{1}{e}\boldsymbol{\nabla}\cdot[\mathbf{J}(\mathbf{r},t)]$$

- Approximate dispersion relation:

$$\omega(q) \simeq \sqrt{\# v_F^2 q^2 + \frac{2\pi e^2 n_0 v_F^2}{\epsilon_b \varepsilon_F} q + \omega_c(\varepsilon_F)^2}$$

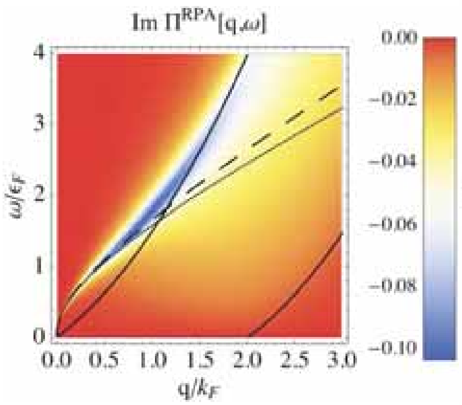
$$\# = \begin{cases} 1/2 & \text{(dotted line) Hydrodynamic} \\ 3/4 & \text{(dashed line) RPA} \end{cases}$$

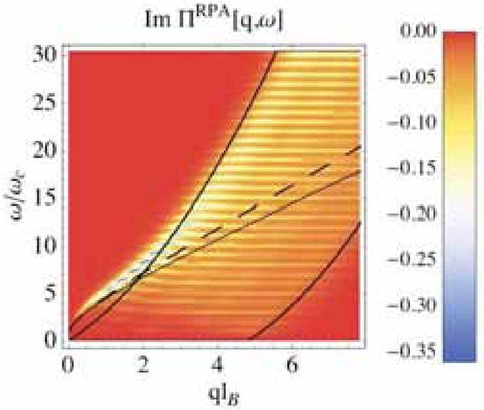


Dispersion of the plasmon and upper hybrid modes

B = 0 | B ≠ 0

2DEG



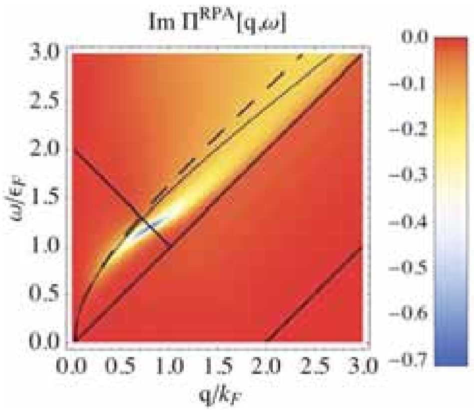


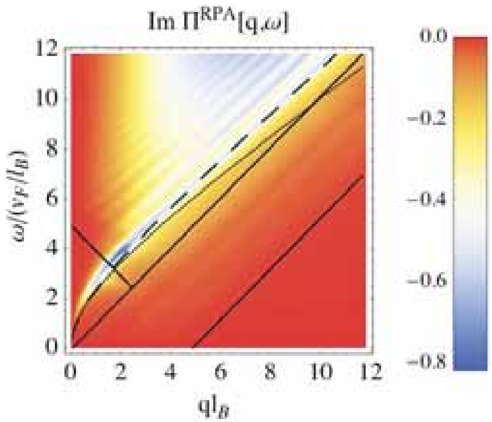
- Approximate dispersion relations

$$\omega(q) \simeq \sqrt{\# v_F^2 q^2 + \frac{2e^2\varepsilon_F}{\epsilon_b} q + \omega_c^2}$$

$$\# = \begin{cases} 1/2 & \text{Hydrodynamic (dotted line)} \\ 3/4 & \text{RPA (dashed line)} \end{cases}$$

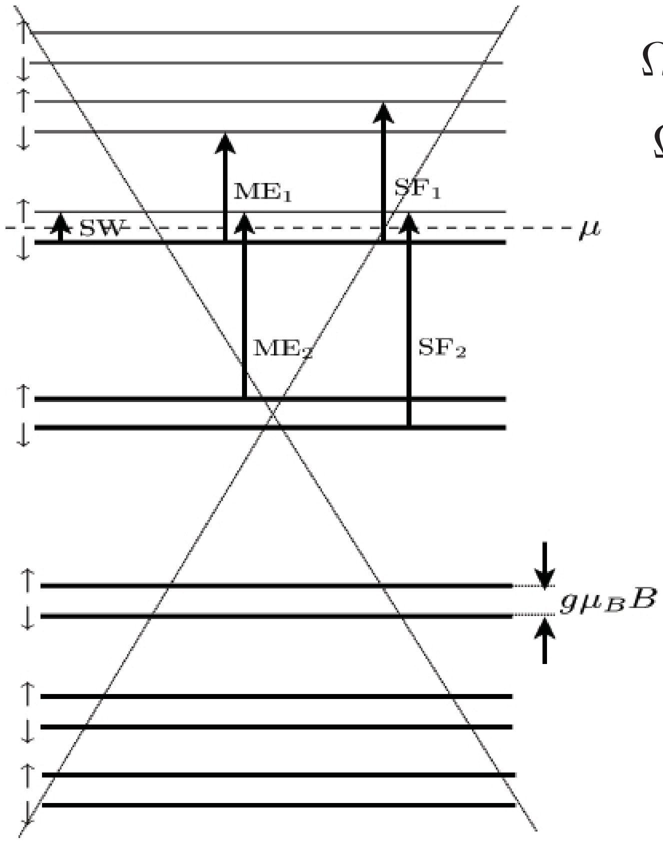
Graphene





$$\omega(q) \simeq \sqrt{\# v_F^2 q^2 + \frac{2\pi e^2 n_0 v_F^2}{\epsilon_b \varepsilon_F} q + \omega_c(\varepsilon_F)^2}$$

Spin-flip & spin-wave modes in graphene



Time-dependent Hartree-Fock approximation

$$\begin{aligned}\Omega_{ME}(q) &= E_{kin}^{(n_e,n_h)} + \Delta E^{(n_e,s_z^e;n_h,s_z^h)}(q)\\ \Omega_{SW}(q) &= g\mu_B B S_z + \Delta E^{(n_e,s_z^e;n_h,s_z^h)}(q)\\ \Omega_{SF}(q) &= E_{kin}^{(n_e,n_h)} + g\mu_B B S_z + \Delta E^{(n_e,s_z^e;n_h,s_z^h)}(q)\end{aligned}$$

$$E_{kin}^{(n_e,n_h)} = \frac{v_{\rm F}}{l_B}\sqrt{2}\,(\lambda_e\sqrt{n_e} - \lambda_h\sqrt{n_h})$$

Includes:

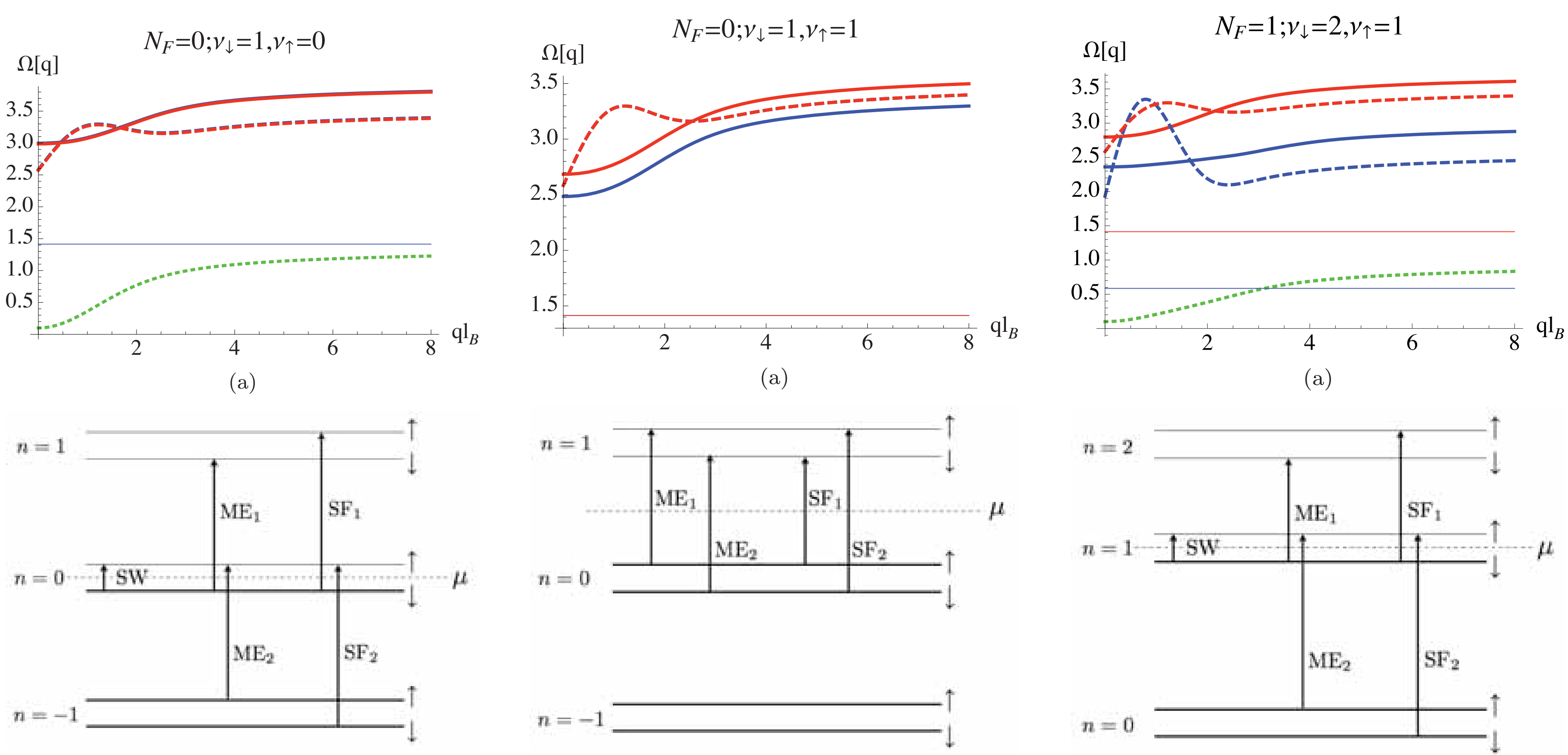
-Depolarization or exchange term (RPA)

-Direct Coulomb interaction between electron and hole (vertex correction)

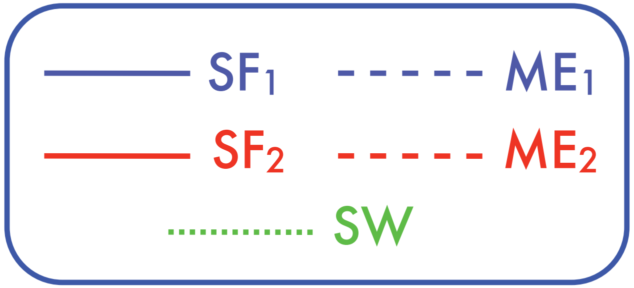
-Difference in exchange self-energy between electron and hole

A. Iyengar, J. Wang, H. A. Fertig, and L. Brey, *PRB* **75**, 125430 (2007); Y. A. Bychkov and G. Martinez, *PRB* **77**, 125417 (2008); RR, J.-N. Fuchs and M. O. Goerbig, *PRB* **82**, 205418 (2010)

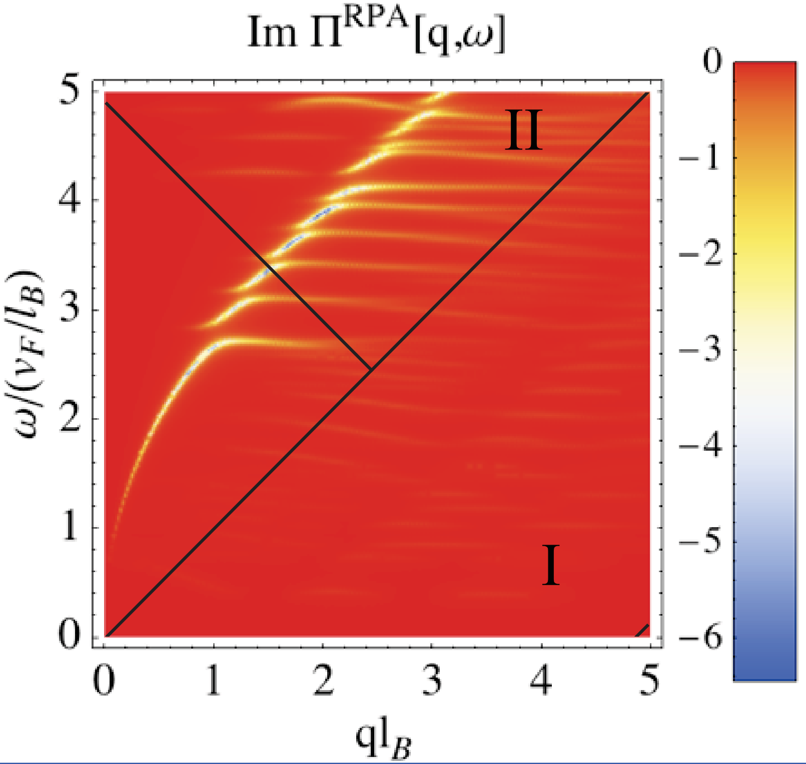
Spin-flip & spin-wave modes in graphene



Konh's theorem does not apply to graphene,
but Larmor's theorem does.



Bernstein modes (BMs): hybridization of a plasmon mode with inter-Landau level transitions



BMs were studied first by I. B. Bernstein, *Phys. Rev.* **109**, 10 (1958)

and later, in the context of a 2DEG, by a number authors:

A. V. Chaplik and D. Heitmann, *J. Phys. C: Solid State Phys.* **18**, 3357 (1985)
G. Gumbs and N.J.M. Horing, *Phys. Rev. B* **31**, 4009 (1985); *ibid* **59**, 2261 (1999)

Phenomenological model for Bernstein modes in graphene

Hamiltonian for the plasmon (UH mode):

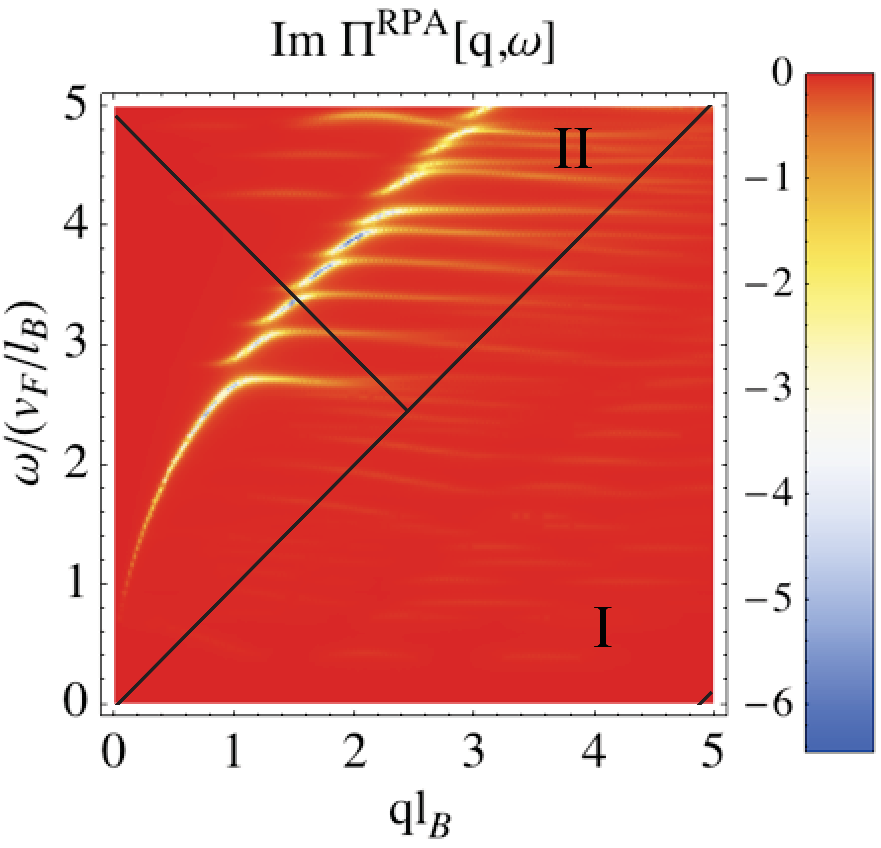
$$H_{\rm uh} = \sum_{\mathbf{q}} \omega_{\rm uh}(q, B) b^\dagger_{\mathbf{q}} b_{\mathbf{q}} \qquad b^{(\dagger)}_{\mathbf{q}} \propto \rho_{uh}(\mathbf{q})$$

Hamiltonian for coupling, via Coulomb interaction, between plasmon and inter-LL excitations:

$$H_{\rm coupl} = \frac{1}{4} \sum_{\mathbf{q}} \frac{2\pi e^2}{\epsilon |\mathbf{q}|} \left[\rho(-\mathbf{q})\rho_{\rm uh}(\mathbf{q}) + \rho_{\rm uh}(-\mathbf{q})\rho(\mathbf{q})\right]$$

Density components of inter-LL excitations:

$$\rho(\mathbf{q}) = \sum_{\lambda n, \lambda' n'} \mathcal{F}_{\lambda n, \lambda' n'}(\mathbf{q}) \sum_{m,m'} \langle m | e^{-i\mathbf{q}\cdot\mathbf{R}} | m' \rangle c^\dagger_{\lambda n, m} c_{\lambda' n', m'}$$



Phenomenological model for Bernstein modes in graphene

Dyson-type equation:

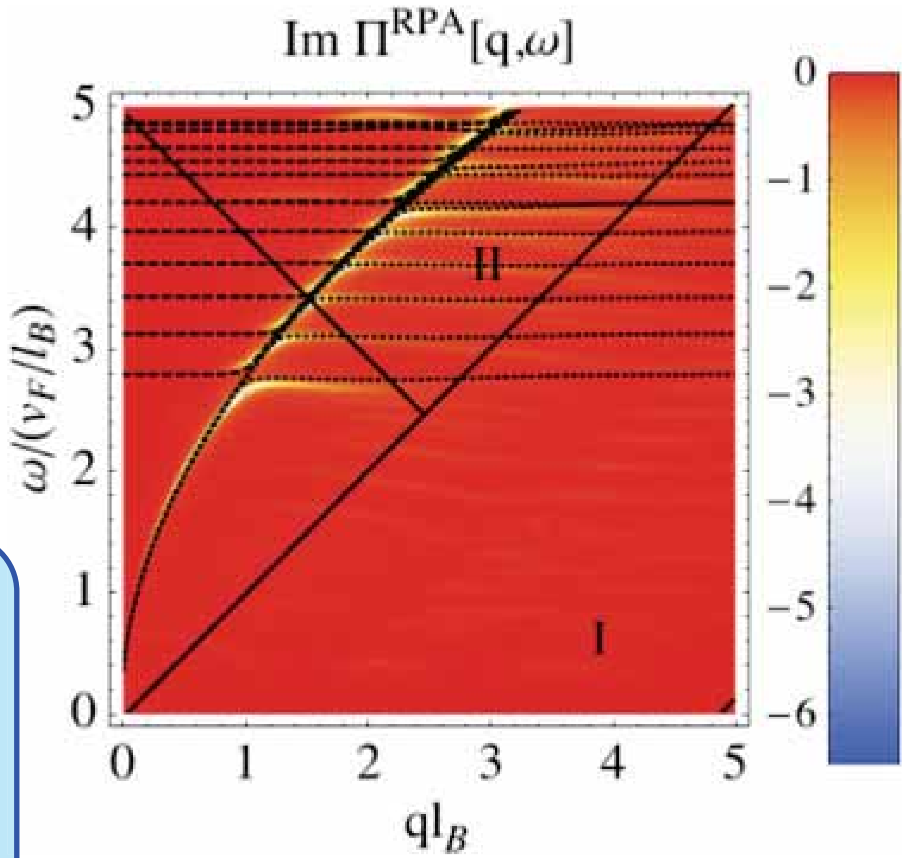
$$[\omega^2 - \omega_{\rm uh}^2][\omega^2 - \Omega_{\lambda n,n'}^2] = \frac{g\mathcal{V}^2}{4}\omega_{\rm uh}\Omega_{\lambda n,n'}$$

$$\mathcal{V} \equiv (e^2/\epsilon q l_B^2)|\mathcal{F}_{\lambda n,n'}(\mathbf{q})|^2$$

$$\Omega_{\lambda n,n'} = \frac{v_F}{l_B}\sqrt{2}(\sqrt{n'} - \lambda\sqrt{n})$$

The poles of the dressed propagator leads to the dispersions:

$$\omega_\pm^2 = \frac{\omega_{\rm uh}(q)^2 + \Omega_{\lambda n,n'}^2}{2} \pm\sqrt{\frac{\left[\omega_{\rm uh}^2 - \Omega_{\lambda n,n'}^2\right]^2}{4} + \frac{g\mathcal{V}^2}{4}\omega_{\rm uh}\Omega_{\lambda n,n'}}$$



Avoided level crossing dominated by resonant term: $\omega \simeq \omega_{\rm uh} \simeq \Omega_{\lambda n,n'}$

Phenomenological model for Bernstein modes in graphene

Linearizing at $\omega_{uh}(q) \simeq \Omega_{\lambda n,n'}$

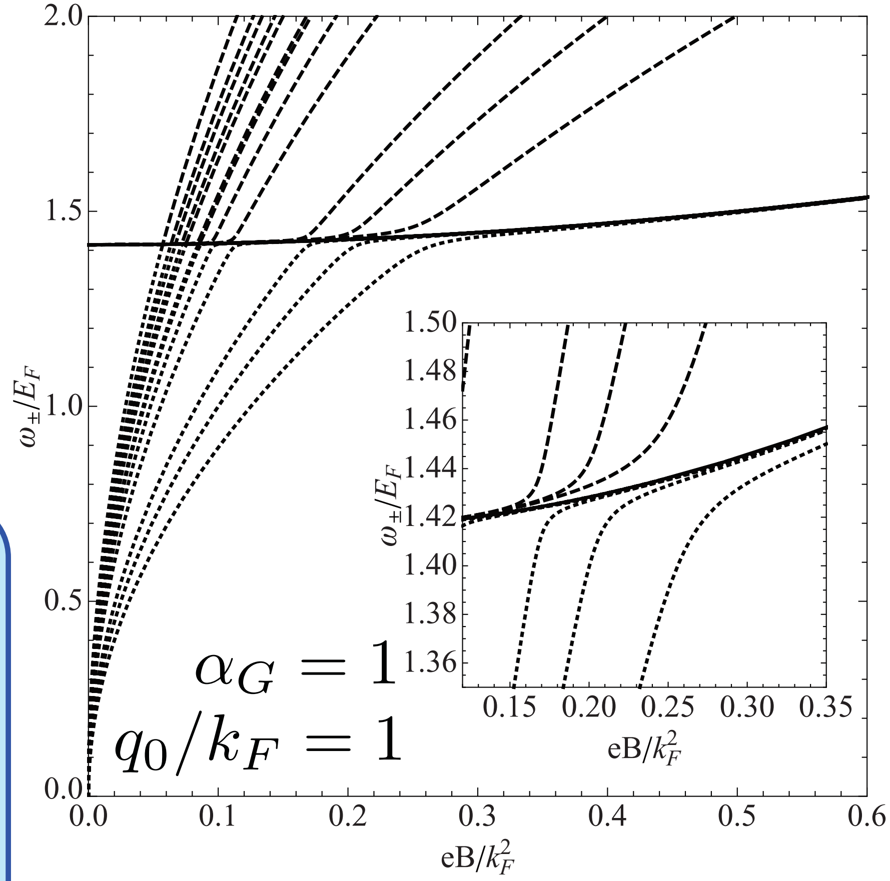
$$\frac{\omega_\pm}{v_F/l_B} \simeq (\sqrt{2n'} - \lambda\sqrt{2n}) \pm \frac{\delta_{\lambda n,n'}(q)}{2}$$

Splitting parameter

$$\delta_{\lambda n,n'}(q) = \frac{\sqrt{g}\mathcal{V}}{v_F/l_B} = \sqrt{g}\frac{\alpha_G}{q l_B}|\mathcal{F}_{\lambda n,n'}(\mathbf{q})|^2$$

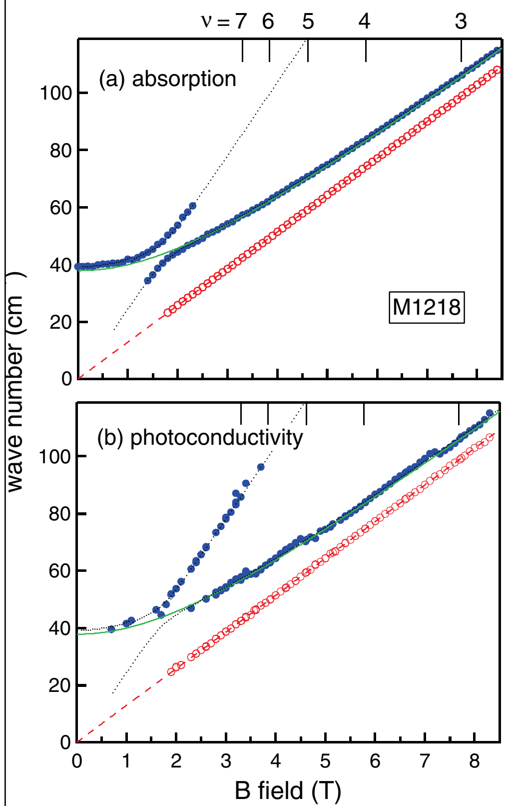
Bernstein modes are expected at the field values

$$(eB)^{-1} \simeq \frac{(\sqrt{n'} - \lambda\sqrt{n})^2}{2\alpha_G q_0 k_F} + \sqrt{\frac{(\sqrt{n'} - \lambda\sqrt{n})^4}{4\alpha_G^2 q_0^2 k_F^2} + \frac{1}{2\sqrt{2}\alpha_G q_0 k_F^3}}$$

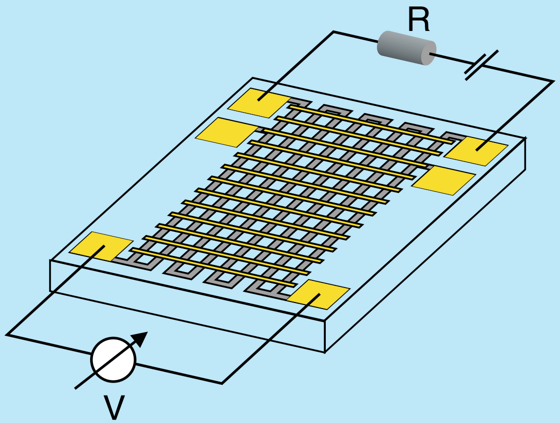


$$\alpha_G = \frac{1}{2\sqrt{\pi n_{\rm el}} q_0}\frac{\omega_{\rm uh}^2(q_0, B \to 0)}{v_F^2}$$

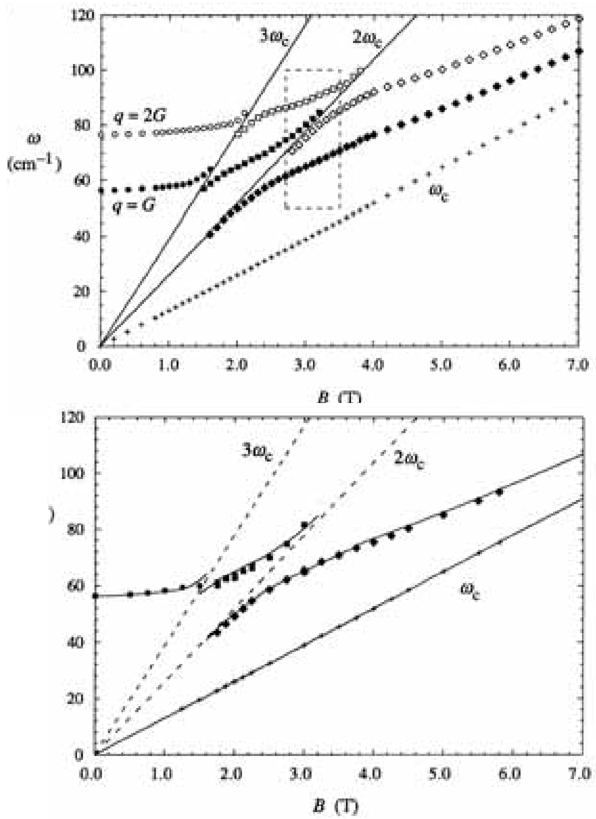
Experimental measurements of Bernstein modes in a 2DEG I: transmission spectroscopy



Hall bar with Ohmic contacts and a grated coupler with a well defined periodicity a



$$q_0 = 2\pi/a$$

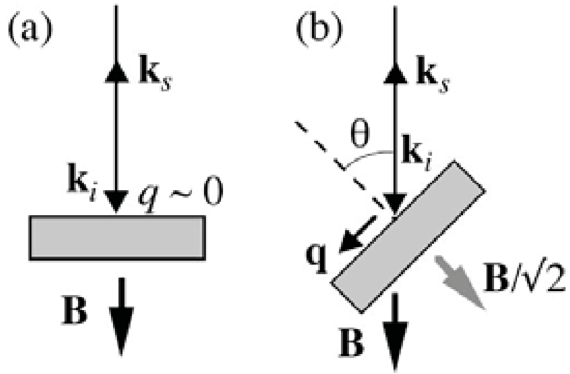


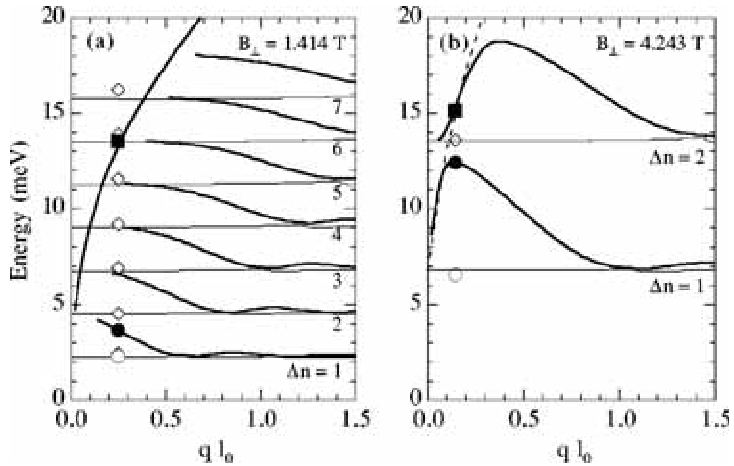
S. Holland, Ch. Heyn, D. Heitmann, E. Batke, R. Hey, K. J. Friedland, and C.-M. Hu *PRL* **93**, 186804 (2004)

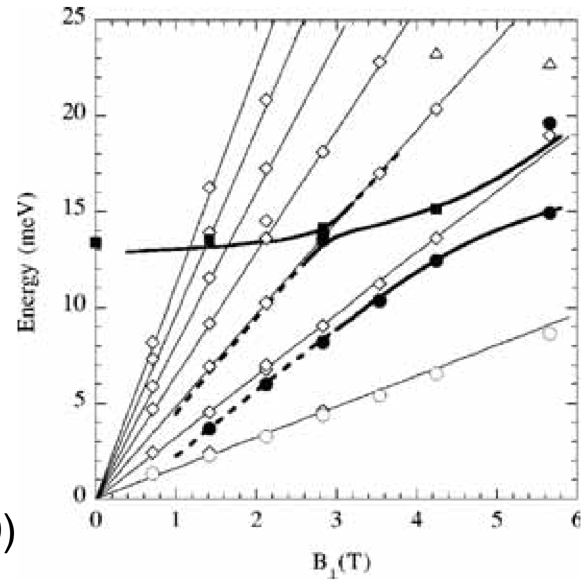
E. Batke, D. Heitmann, J. P. Kotthaus and K. Ploog, *PRL* **54**, 2367 (1985)

D. E. Bangert, R. J. Stuart, H. P. Hughes, D. A. Ritchie and J. E. F. Frost, *Semicond. Sci. Technol.* **11**, 352 (1996)

Experimental measurements of Bernstein modes in a 2DEG II: inelastic light scattering







Angle-resolved Raman scattering for both zero and finite in-plane wave vector transfer to the 2DEG

$$q_0 = \Delta\omega \sin\theta / c$$

$$\Delta\omega = (\omega_i - \omega_s)$$

D. Richards, *PRB* **61**, 7517 (2000)

Summary of the PHES with B≠0

• Study of the particle-hole excitation spectrum of graphene in a magnetic field as compared to the standard 2DEG.

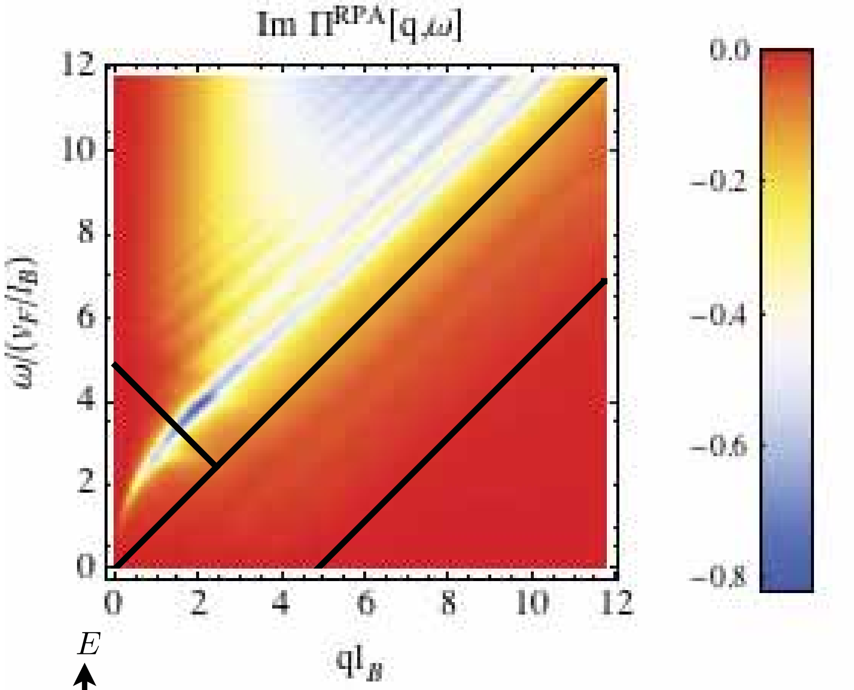
• Landau level quantization yields *linear magneto-plasmon* modes in contrast to the 2DEG, where the equidistant LL structure leads to pronounced horizontal magneto-exciton modes.

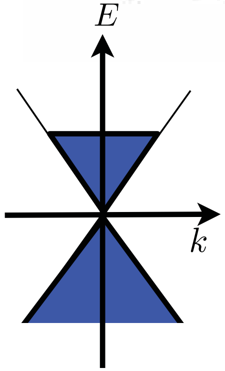
• Inelastic (Raman) light scattering could be used to reveal the existence and measure the dispersion relation of the linear magneto-plasmons, as well as the Bernstein modes in graphene.

	2DEG	Doped graphene
$B = 0, r_s = 0$	Continuum PHES	Intra- & inter-band continuum PHES
$B = 0, r_s \neq 0$	Continuum PHES + Plasmon	Intra- & inter-band continuum PHES + Plasmon
$B \neq 0, r_s = 0$	Discretized PHES (hints magneto-excitons)	Discretized PHES (hints linear magneto-plasmons)
$B \neq 0, r_s \neq 0$	Discretized PHES (ME) + upper hybrid mode	Discretized PHES (LMP) + upper hybrid mode

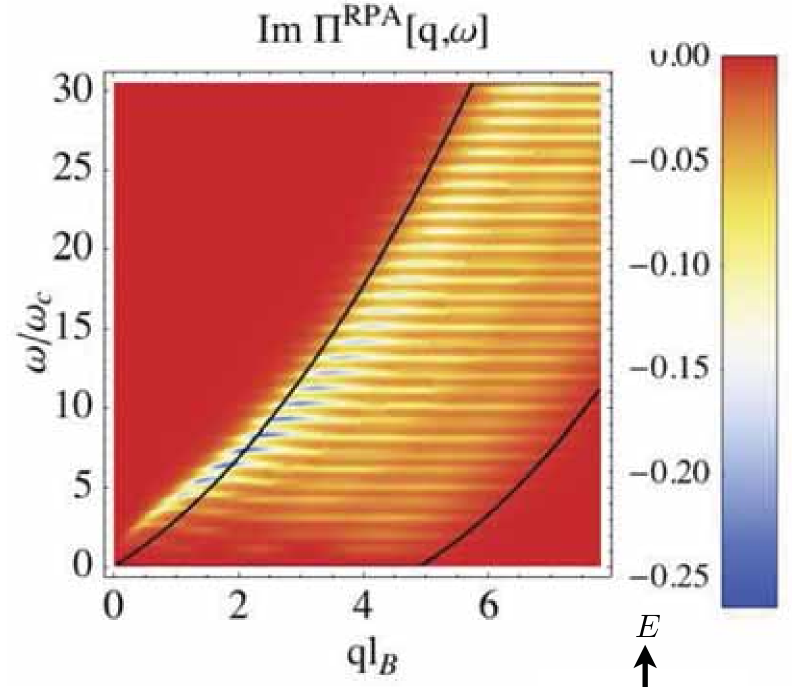
RR, J.-N. Fuchs and M.O. Goerbig, *Phys. Rev. B* **80**, 085408 (2009); *ibid* **82**, 205418 (2010); *ibid* **83**, 205406 (2011)
RR, M.O. Goerbig & J.-N. Fuchs, *Semicond. Sci. Technol.* **25**, 034005 (2010) [short review paper]
S. Yuan, **RR**, & M. I. Katsnelson, *Solid State Comm.* **152**, 1446 (2012)

Conclusion: linear magneto-plasmons rather than horizontal magneto-excitons

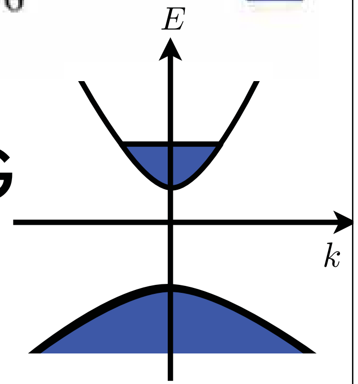




Doped graphene



Standard 2DEG



RR, J.-N. Fuchs and M.O. Goerbig, *Phys. Rev. B* **80**, 085408 (2009); *ibid* **82**, 205418 (2010); *ibid* **83**, 205406 (2011)
RR, M.O. Goerbig & J.-N. Fuchs, *Semicond. Sci. Technol.* **25**, 034005 (2010) [short review paper]

CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

ICMM

Many thanks to...

Shengjun Yuan & Misha Katsnelson

Radboud University of Nijmegen. The Netherlads.

Jean-Noël Fuchs & Mark O. Goerbig

Laboratoire de Physique des Solides. Université Paris-Sud. France.

...and to you for your attention

Tainan, July 2012

Happy Birthday Prof. Lin!!!