Effects of a potential barrier on spin currents along a nanotube

1st Workshop on Nanoscience: Graphene
National Cheng Kung University
Tainan City, Taiwan, 17 December, 2011

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Overview

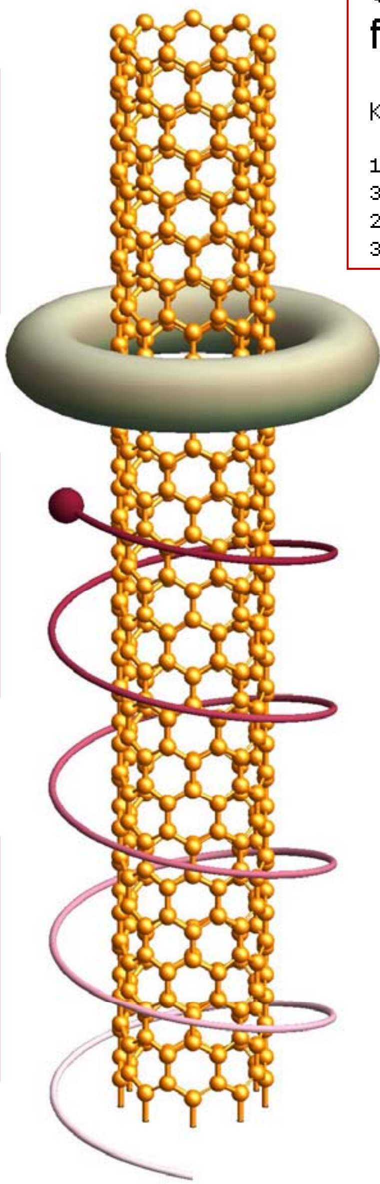
- **Consider effects of the spin orbit interaction (SOI) and disorder on electrons moving along the surface of a nanoube**
- **Obtain analytic expressions for the spin-split energy bands**
- **Calculate scattering amplitude from a potential barrier located around the axis of the nanotube into spin-dependent states**
- **Estimate phenomenologically the effect of disorder within the potential barrier on the transmission probability**
- **Analyze the relative role of SOI and disorder**
- **Prove that transmission probability**
  - **depends on the linear and angular momentum of the incoming particle and its spin orientation**
  - **is reduced by disorder**
- **Demonstrate that in the presence of disorder perfect transmission may not be achieved for finite barrier heights**

Experimentally observed SOI in gated nanotubes

Electron confined to move along the surface of a nanotube with gate voltage applied perpendicular to the surface of the nanotube

Coupling between spin and orbital angular momentum

Observable influence of the SOI on the transport properties of carbon nanotubes



Coherent transport of electron spin in a ferromagnetically contacted carbon nanotube

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Nature **401**, 572-574 (7 October 1999)

nature

Appl. Phys. Lett. **80**, 3144 (2002); doi:10.1063/1.1471570 (3 *pages*)

AIP Applied Physics Letters

Spin-coherent transport in ferromagnetically contacted carbon nanotubes

B. Zhao, I. Mönch, H. Vinzelberg, T. Mühl, and C. M. Schneider

Institute of Solid State and Materials Research IFW Dresden, P.O. Box 270 016, D-01171 Dresden, Germany

Local Gate Control of a Carbon Nanotube Double Quantum Dot

N. Mason,*† M. J. Biercuk,* C. M. Marcus†

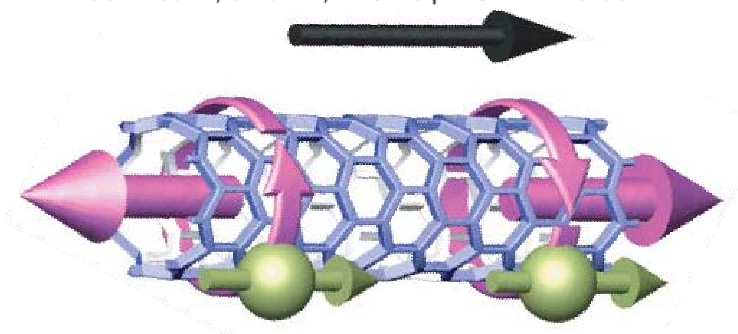
Science AAAS

303, 655 (2004)

Experimentally observed SOI in gated nanotubes

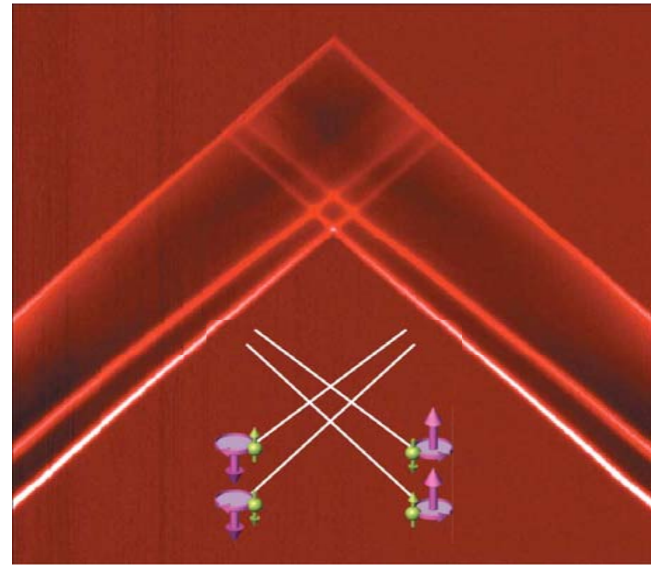
Coupling of spin and orbital motion of electrons in carbon nanotubes

F. Kuemmeth[1]*, S. Ilani[1]*, D. C. Ralph[1] & P. L. McEuen[1]



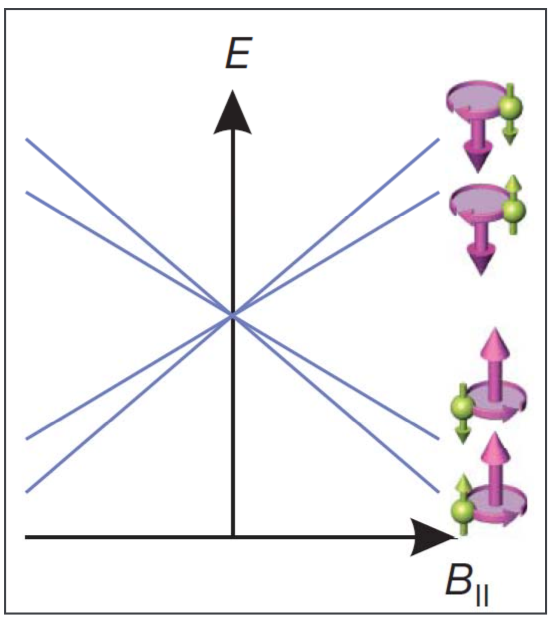
nature
LETTERS
Vol 452 | 27 March 2008

Measured excitation spectrum

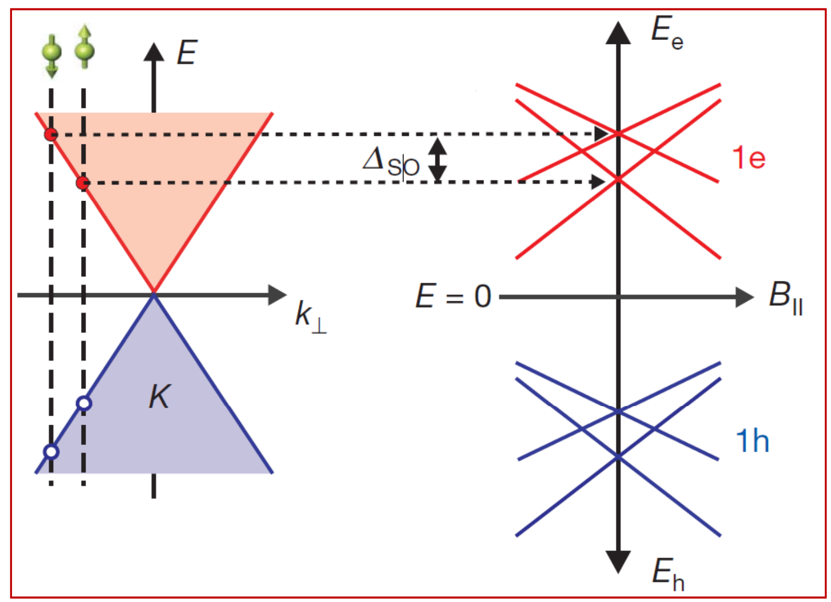


States split into two at $B = 0$

Predicted excitation spectrum



Separation between
- upper states decreases with increasing *B*
- lower states increases with increasing *B*



Rashba-Bychkov SOI for electrons confined to move along a nanotube

PHYSICAL REVIEW B **73**, 165315 (2006)

Effect of spontaneous spin depopulation on the ground-state energy of a two-dimensional spintronic system

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(Received 17 October 2005; revised manuscript received 8 February 2006; published 17 April 2006)

SOI on the nanotube is of Rashba-Bychkov type

Spin-orbit Hamiltonian arising from electrostatic confinement

$$H_{SO} = \frac{\hbar^2}{2m^{*2}c^2}\left[\vec{\nabla}V(\vec{r})\times\vec{p}\right]\cdot\vec{\sigma}$$

$\vec{\nabla}V(\vec{r}) \equiv$ perpendicular potential gradient

$\vec{\sigma} \equiv$ vector of Pauli spin matrices

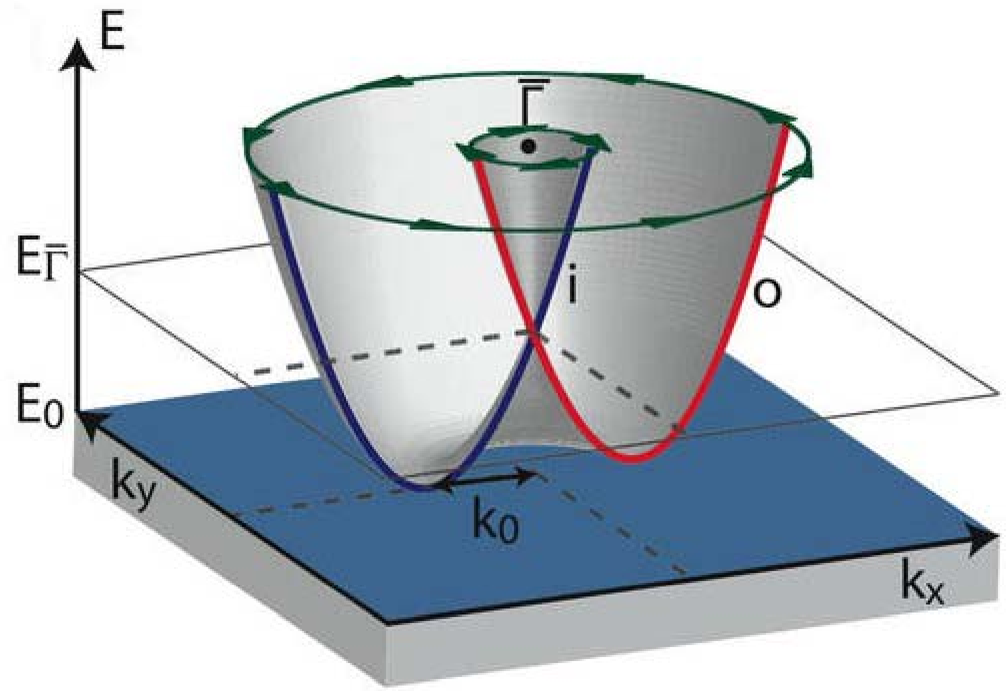
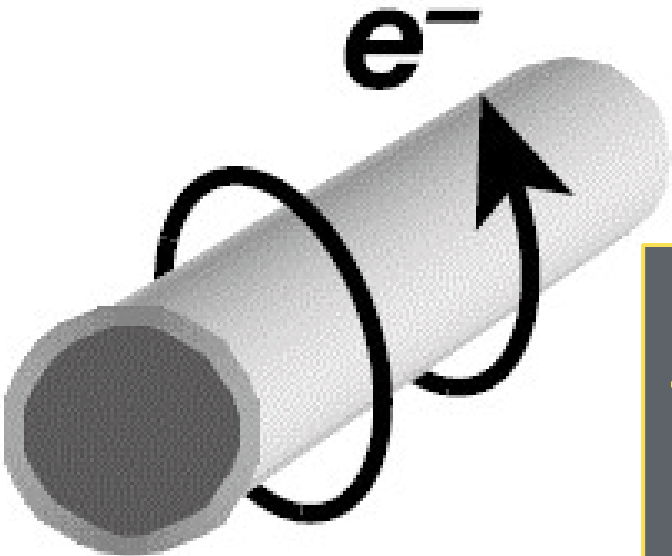


Image Credit: J. Hugo Dil, Physik-Institut, Universität Zürich



Model Hamiltonian

Employing a continuum model, obtain analytic expressions for the spin-split energy bands of electrons moving along the surface of a nanotube using the Hamiltonian

$$H = \frac{1}{2m^*}\left(\hat{p}_z^2 + \hat{p}_\phi^2\right) + \frac{\alpha_R}{\hbar}\left[\left(\sigma_1 \sin\phi - \sigma_2 \cos\phi\right)\hat{p}_z + \sigma_3 \hat{p}_\phi\right]$$

where

$m^* \equiv$ effective mass of the electron

$\alpha_R \equiv$ Rashba SOI parameter

$\sigma_i \equiv$ Pauli spin matrices

$\phi \equiv$ azimuth angle measured from the nanotube symmetry axis

2007 *J. Phys.: Condens. Matter* **19** 106213

Plasma excitations for cylindrical nanotubes with spin splitting

Godfrey Gumbs, Yonatan Abranyos and Tibab McNeish

Journal of Physics: Condensed Matter

Physical Review A

Phys. Rev. A 70, 050302(R) (2004) [4 pages]

Quantum entanglement for acoustic spintronics

Godfrey Gumbs and Yonatan Abranyos

Eigenfunctions

Two-dimensional spinor wavefunctions

$$\left| u_\nu(\phi, z) \right\rangle = \begin{pmatrix} u_\nu^+(\phi, z) \\ u_\nu^-(\phi, z) \end{pmatrix}$$

Traveling wave-like solutions in the *z*-direction

$$u_\nu^\pm(\phi, z) = \frac{e^{ikz}}{\sqrt{L_z}} \Phi^\pm(\phi)$$

SOI mixes subbands

$$\Phi^\pm(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} c_l^\pm(k_z) e^{il\phi}$$

where

$L_z \equiv$ normalization length

$l \equiv$ angular momentum quantum number labels each subband

$c_l \equiv$ expansion coefficients

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Plasma excitations for cylindrical nanotubes with spin splitting

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Spin-split energy eigenvalues in the absence of a barrier

Eigenenergies given by

$$\varepsilon^{s}\left(k_z,l,\alpha_R\right)=\frac{1}{2}\left[E_{l+1}^{2}\left(k_z,\alpha_R\right)+E_{l}^{1}\left(k_z,\alpha_R\right)\right]+\frac{s}{2}\sqrt{\left[E_{l+1}^{2}\left(k_z,\alpha_R\right)+E_{l}^{1}\left(k_z,\alpha_R\right)\right]^{2}+4\alpha_R^2k_z^2}$$

where

$$E_l^1\left(k_z,\alpha_R\right)=E^{(0)}\left(k_z,l\right)-l\varepsilon_\alpha$$

and

$$E_l^2\left(k_z,\alpha_R\right)=E^{(0)}\left(k_z,l\right)+l\varepsilon_\alpha$$

with

$$\varepsilon_\alpha=\frac{\alpha_R}{R}$$

$$E^{(0)}\left(k_z,l\right)=\frac{\hbar^2k_z^2}{2m^*}+l^2\varepsilon_R$$

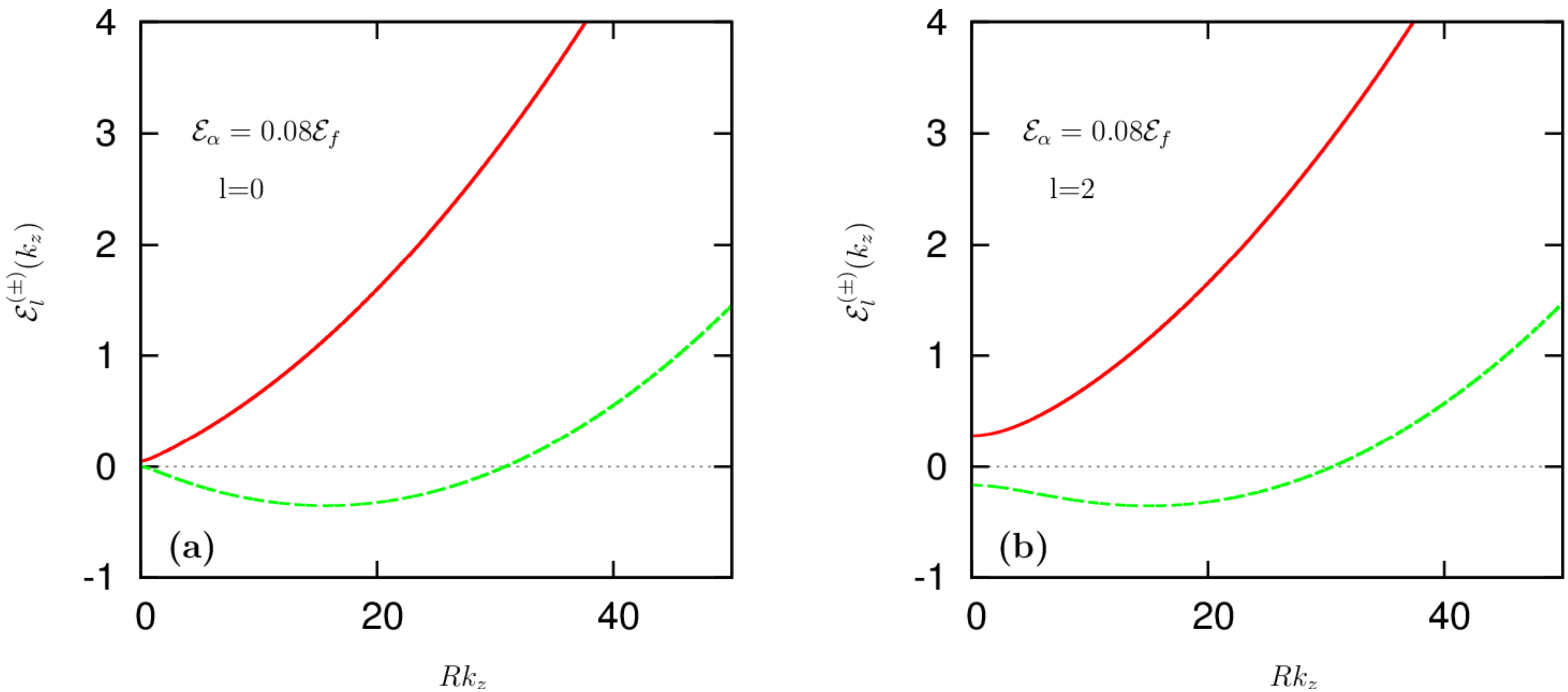
with

$$\varepsilon_R=\frac{\hbar^2}{2m^*R^2}$$

$s=\pm$ indicates two spin orientations

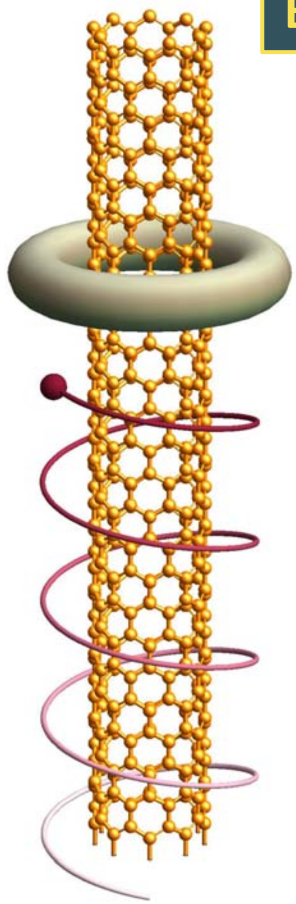
$R\equiv$ radius of the nanotube

Energy dispersion in the absence of a barrier



Energy eigenvalues as a function of k_z in the presence of SOI. Red line for the "+" state. Green line for the "–" state.

Energy eigenvalues in the presence of a barrier



Cylindrical potential barrier of height *U* and width *w*.

Eigenenergies given by

$$\varepsilon^{s}\left(k_z,l,\alpha_R\right)-U=\bar{\varepsilon}^{s}\left(k_z^b,l,\alpha_R\right)$$
$$=\frac{1}{2}\left[E_{l+1}^{2}\left(k_z^b,\alpha_R\right)+E_l^1\left(k_z^b,\alpha_R\right)\right]$$
$$+\frac{s}{2}\sqrt{\left[E_{l+1}^{2}\left(k_z^b,\alpha_R\right)+E_l^1\left(k_z^b,\alpha_R\right)\right]^2+4\alpha_R^2\left(k_z^b\right)^2}$$

where $E_l^1\left(k_z,\alpha_R\right)=E^{(0)}\left(k_z,l\right)-l\varepsilon_\alpha$

$E_l^2\left(k_z,\alpha_R\right)=E^{(0)}\left(k_z,l\right)+l\varepsilon_\alpha$ **with** $\varepsilon_\alpha=\frac{\alpha_R}{R}$

and $E^{(0)}\left(k_z,l\right)=\frac{\hbar^2k_z^2}{2m^*}+l^2\varepsilon_R$ **with** $\varepsilon_R=\frac{\hbar^2}{2m^*R^2}$

As the height of the barrier increases, eigenvalue equation cannot be satisfied by real k_z^b

$$k_z^b\rightarrow\pm i\kappa$$

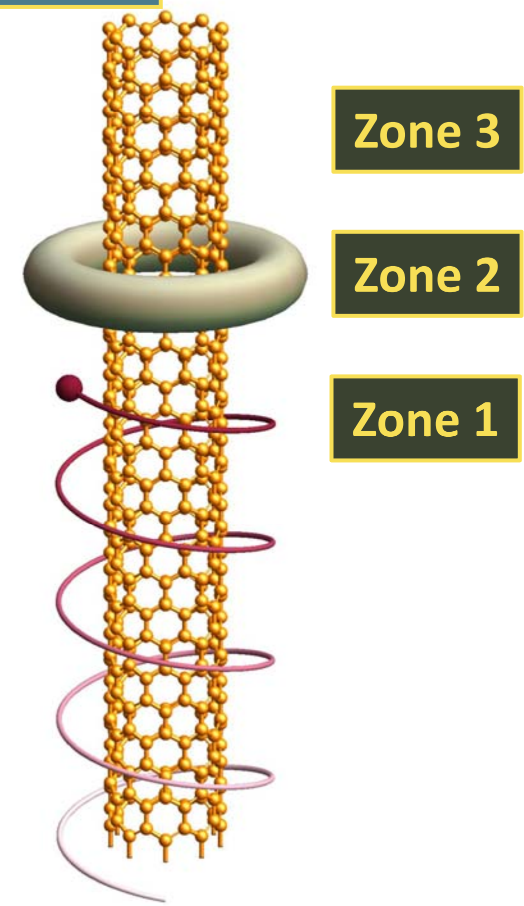
Reflection and transmission amplitudes

Wave functions in the three regions of interest

$$\left|\psi^{1}_{k_z^i,L^i}\right\rangle = a_+\left|k_z^i,L^i,+\right\rangle + a_-\left|k_z^i,L^i,-\right\rangle + \sum_{ss'=\pm} r_{ss'}\left|-k_z^i,L^R,ss'\right\rangle$$

$$\left|\psi^{2}_{k_z^b,L^b,s}\right\rangle = \sum_{rs'=\pm} b_{rs'}\left|k_z^b,L^b,rs'\right\rangle$$

$$\left|\psi^{3}_{k_z^t,L^t,s}\right\rangle = \sum_{ss'=\pm} t_{ss'}\left|k_z^t,L^t,ss'\right\rangle$$



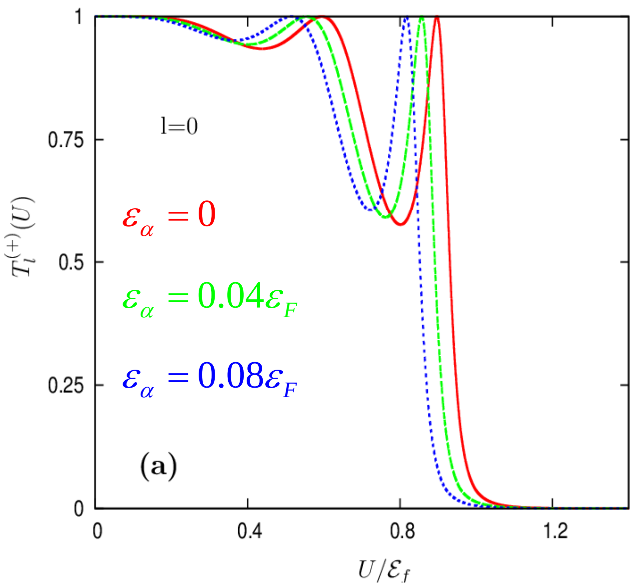
Transmission probability for spin up electrons

$$T^+(l) = \left|t_{++}(l)\right|^2 + \left|t_{+-}(l)\right|^2$$

Conservation of angular momentum

$$L^i = L^b = L^t$$

Tunneling through a potential barrier



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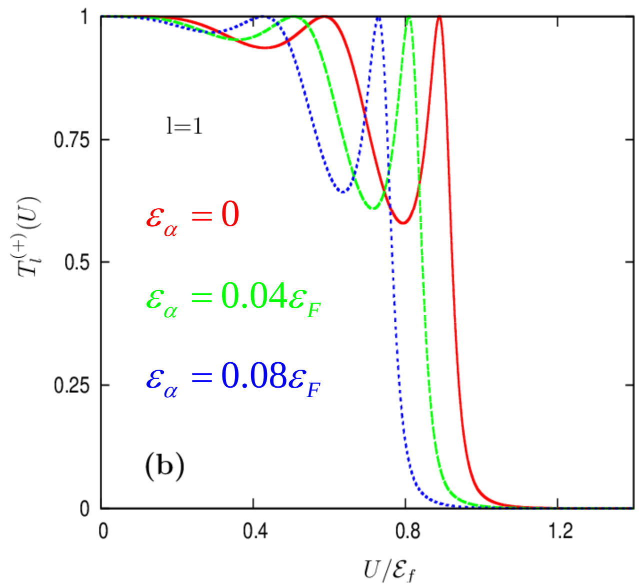

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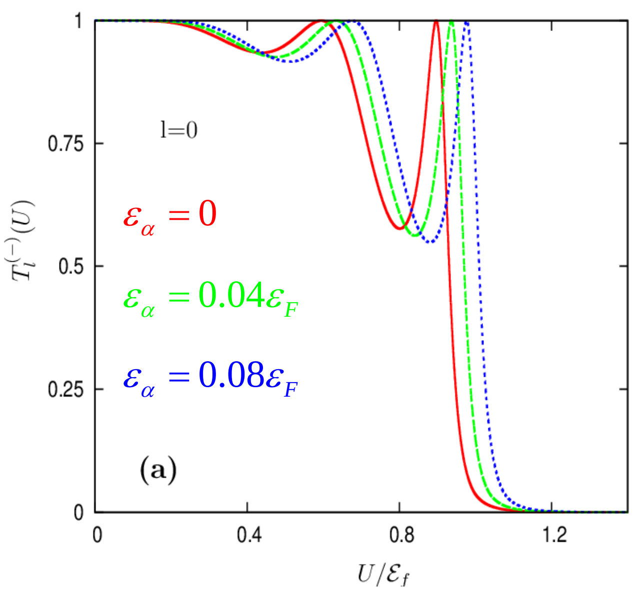

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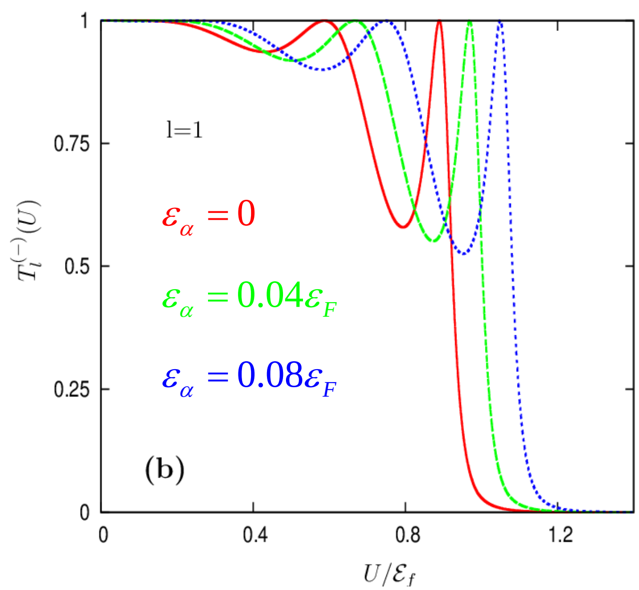

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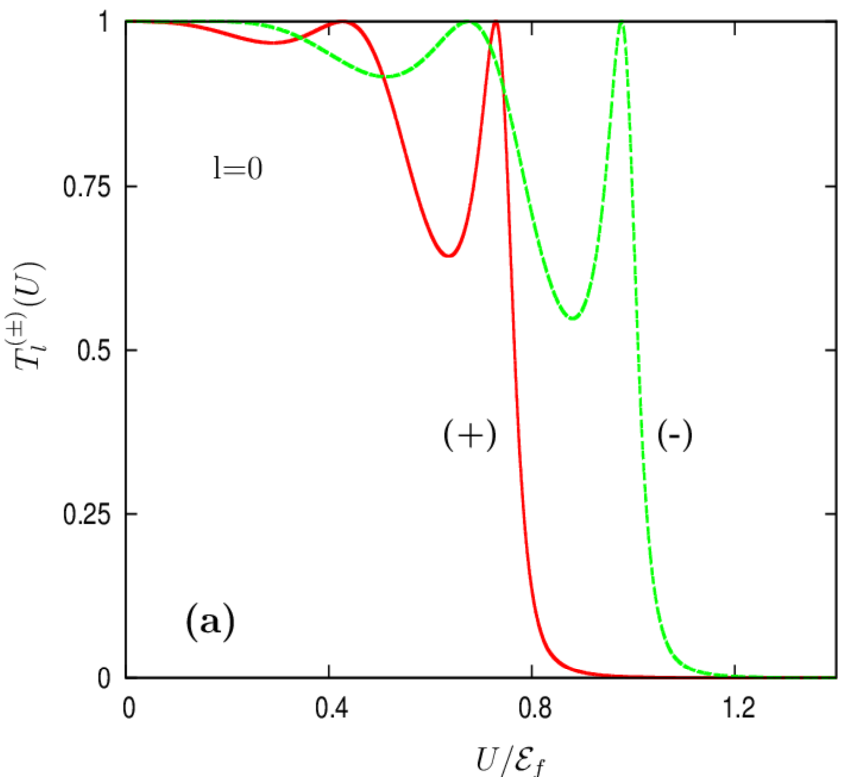
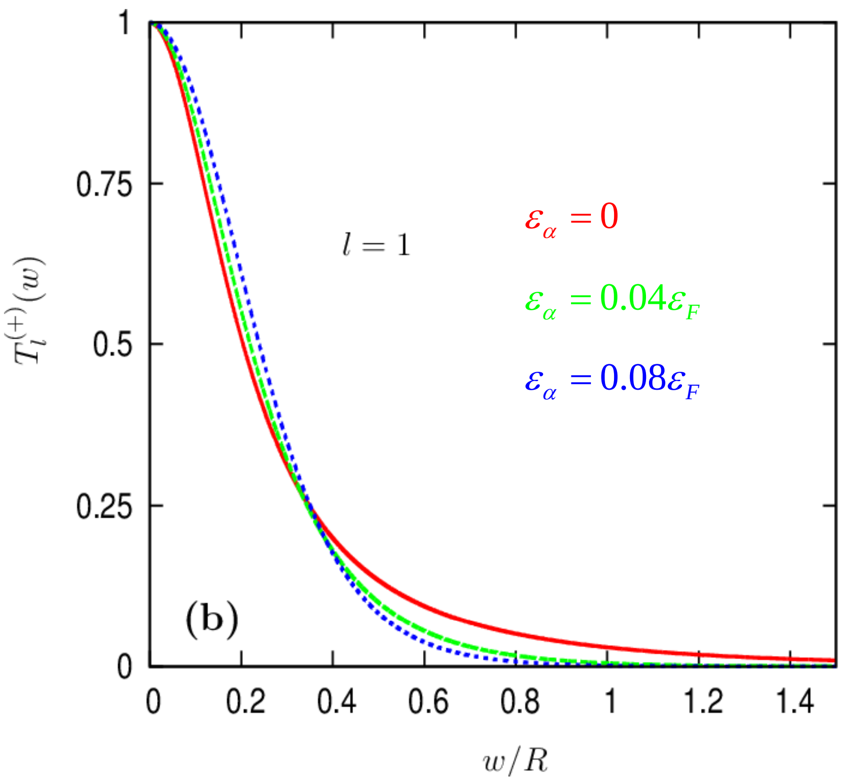
Transmission probability as function of potential barrier height, *U*



$$\varepsilon_\alpha = \frac{\alpha_R}{R} = 0.08\varepsilon_F$$

Transmission probability as function of potential barrier width, *w*



Disorder – lifetime effect

Breakdown of momentum conservation due to impurity scattering introduced as a finite lifetime of the electron states

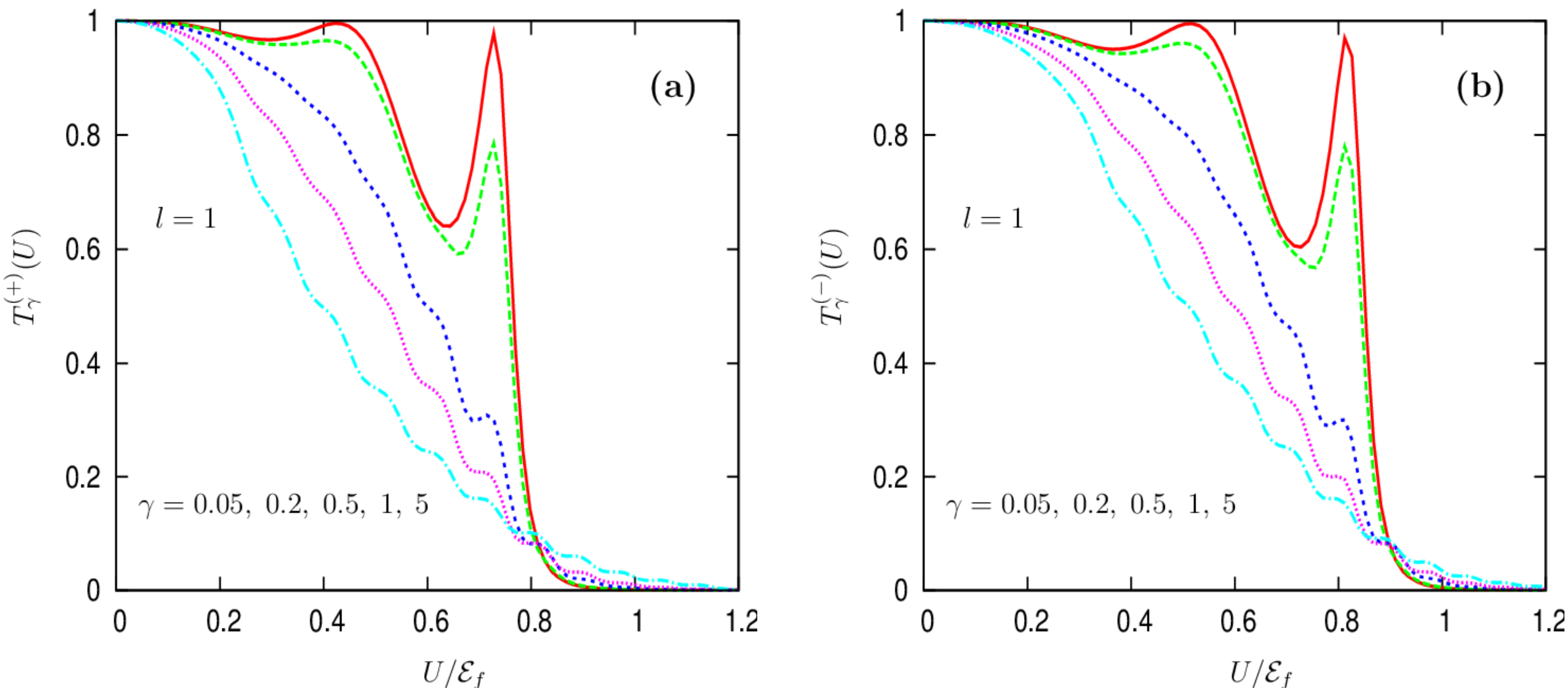
$$t_{s,s'}^{(\text{disorder})}(l) = \frac{1}{\mathcal{N}(\gamma)\gamma} \sum_{l'=-\infty}^{\infty} \int_0^{2\pi} \frac{d\varphi}{2\pi} \int_0^{2\pi} \frac{d\varphi' t_{s,s'}(l')}{2\pi} \frac{e^{i(l-l')(\varphi-\varphi')}}{(\varphi-\varphi')^2 + \frac{1}{\gamma^2}}$$

where

$$\mathcal{N}(\gamma) = \sum_{l'=-\infty}^{\infty} \int_0^{2\pi} \frac{d\varphi}{2\pi} \int_0^{2\pi} \frac{d\varphi'}{2\pi} \frac{e^{i(l-l')(\varphi-\varphi')}}{(\varphi-\varphi')^2 + \frac{1}{\gamma^2}}$$

and $\gamma \equiv$ inverse lifetime

Effects of disorder
Impurity scattering: lifetime effect



$$\varepsilon_\alpha = 0.08\varepsilon_F$$

Disorder – temperature effect

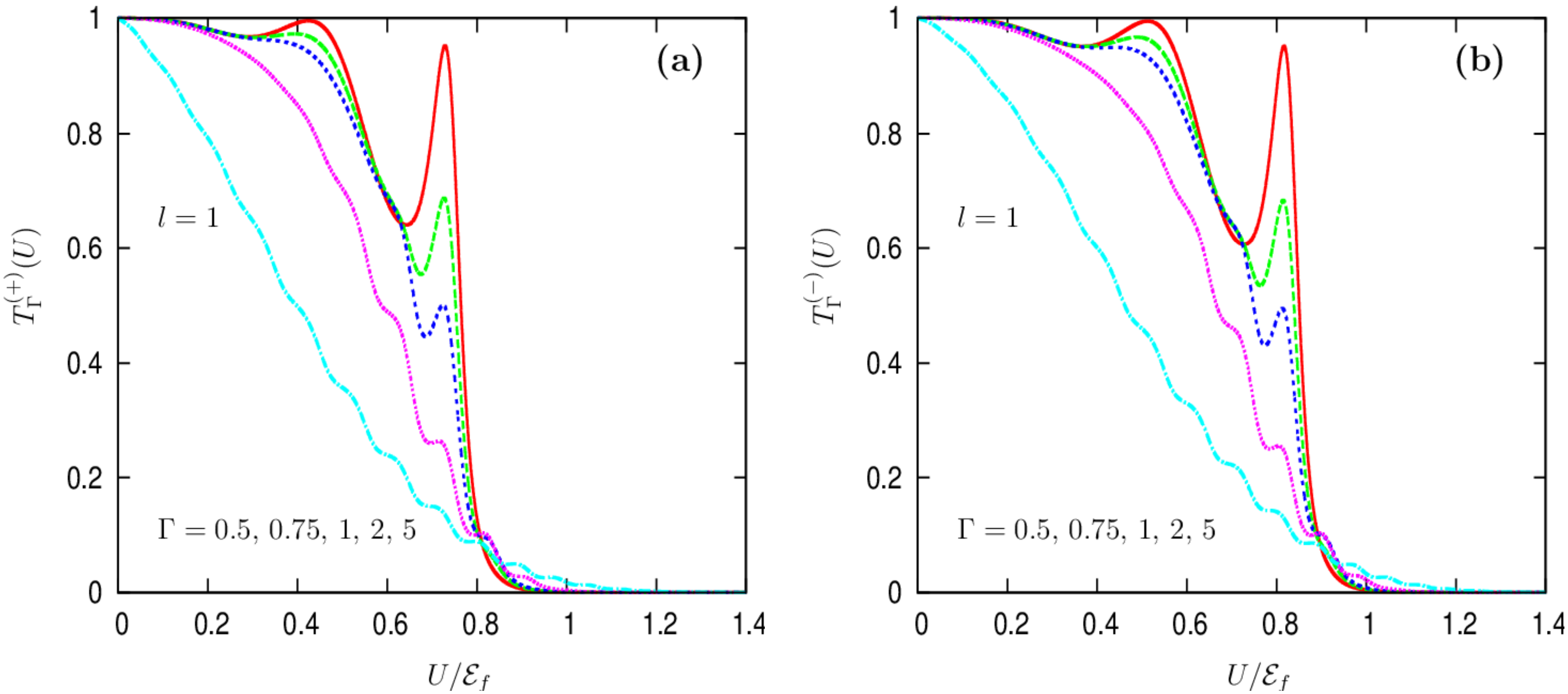
Breakdown of momentum conservation due to a dimensionless temperature parameter

$$t_{s,s'}^{(\text{disorder})}(l) = \frac{\sum_{l'=-\infty}^{\infty} t_{s,s'}(l')\, e^{-\frac{(l-l')^2}{\Gamma^2}}}{\sum_{l'=-\infty}^{\infty} e^{-\frac{(l-l')^2}{\Gamma^2}}}$$

where

$\Gamma \equiv$ dimensionless phenomenological temperature

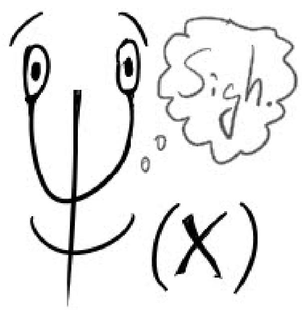
Effects of disorder
Momentum dissipation: thermal effect



$$\varepsilon_\alpha = 0.08\varepsilon_F$$

Conclusions

- Transmission probability exhibits oscillatory behavior
- Barrier height where perfect transmission occurs depends on the strength of the SOI, the angular momentum as well as on the spin orientation
- SOI may be used as filter to obtain unimpeded transport through a specified potential barrier height


Sigh.
(X)