Workshop on Compressed Sensing and Quadratic Programming

July 10-12, 2013
R413, NCTS, NCKU

DATE: 7/10 (WED.)

10:00-12:00	*A Brief Introduction to Compressed Sensing* Joe-Mei Feng Institute for Numerical and Applied Mathematics University of Goettingen
12:00-14:00	Lunch and Break
14:00-16:00	*On RIC bounds of Compressed Sensing Matrices for Approximating Sparse Solutions Using Lq Quasi Norms* Ruey-Lin Sheu Department of Mathematics National Cheng Kung University

DATE: 7/11 (THU.)

10:00-12:00	*A Brief Introduction to Quantization for Compressed Sensing* Joe-Mei Feng Institute for Numerical and Applied Mathematics University of Goettingen
12:00-14:00	Lunch and Break
14:00-15:30	*An SDP Parametric Approach for Solving Single Ratio Quadratic Fractional Programming* Nguyen Van Bong Department of Mathematics National Cheng Kung University
15:30-17:00	*Improved Estimation of Duality Gap in Box Constrained Quadratic Programming Using a Weighted Distance Measure* Ji-Yi Wong Department of Mathematics National Cheng Kung University

DATE: 7/12 (FRI.)

10:00-12:00	*Recent Progress on Mathematical Programming* Discussion Session hosted by Joe-Mei Feng

Website: http://www.ncts.ncku.edu.tw/math/workshop/130710/poster.pdf

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Schedule

July 10, 2013

10:00-12:00 *Speaker*: Joe-Mei Feng
Institute for Numerical and Applied Mathematics, University of Goettingen
fengjoemei@gmail.com
Title: **A Brief Introduction to Compressed Sensing**

12:00-14:00 Lunch and Break

14:00-16:00 *Speaker*: Ruey-Lin Sheu
Department of Mathematics, National Cheng Kung University
rsheu@mail.ncku.edu.tw
Title: **On RIC bounds of Compressed Sensing Matrices for Approximating Sparse Solutions Using ℓ_q Quasi Norms**

July 11, 2013

10:00-12:00 *Speaker*: Joe-Mei Feng
Institute for Numerical and Applied Mathematics, University of Goettingen
fengjoemei@gmail.com
Title: **A Brief Introduction to Quantization for Compressed Sensing**

12:00-14:00 Lunch and Break

14:00-15:30 *Speaker:* Nguyen Van Bong
Department of Mathematics, National Cheng Kung University
bongtnmath@yahoo.com.vn
Title: **An SDP Parametric Approach For Solving Single Ratio Quadratic Fractional Programming**

15:30-17:00 *Speaker*: Ji-Yi Wong
Department of Mathematics, National Cheng Kung University
L16004012@mail.ncku.edu.tw
Title: **Improved Estimation of Duality Gap in Box Constrained Quadratic Programming Using a Weighted Distance Measure**

July 12, 2013

10:00-12:00 Discussion Session *hosted by* Joe-Mei Feng

Topic: Recent Progress on Mathematical Programming

A Brief Introduction to Compressed Sensing

Joe-Mei Feng
Institute for Numerical and Applied Mathematics
University of Goettingen

In many practical problems, such as signal or image processing, people would like to reconstruct a signal from measured data. When the data are measured through a linear process, then it is a problem of solving a linear system.
Most of the natural signals can be represented approximately sparse over proper bases. If we apply a full rank linear system to process the signal, is there any redundancy? If only few measured data are given, to which level can we still recover the signal? To answer these questions, the research on compressed sensing provides the criteria of reconstructing these sparse signals from few linearly measured data.
In this talk, I will introduce the criteria of recovering the signal from only few measured data, and how well some matrices can satisfy the criteria for recovery.

On RIC bounds of Compressed Sensing Matrices for Approximating Sparse Solutions Using ℓ_q Quasi Norms

Reuy-Lin Sheu
Department of Mathematics
National Cheng Kung University

This talk follows the recent discussion on the sparse solution recovery with quasi-norms $\ell_q, q \in (0,1)$ when the sensing matrix possesses a Restricted Isometry Constant δ_{2k} (RIC). Our key tool is an improvement on a version of ``the converse of a generalized Cauchy-Schwarz inequality" extended to the setting of quasi-norm. We show that, if $\delta_{2k} \le \frac{1}{2}$, any minimizer of the ℓ_q minimization, at least for those $q \in (0, 0.9181]$, is the sparse solution of the corresponding underdetermined linear system. Moreover, if $\delta_{2k} \le 0.4931$, the sparse solution can be ecovered by any $\ell_q, q \in (0,1)$ minimization. The values 0.9181 and 0.4931 improve those reported previously in the literature.

A Brief Introduction to Quantization for Compressed Sensing

Joe-Mei Feng
Institute for Numerical and Applied Mathematics
University of Goettingen

Compressed Sensing shows that it is possible to recover a sparse signal from only few measured data. However due to the computer and digital technology, the task is even complicated, while the measured data are further quantized. Therefore the real problem is to reconstruct a signal from quantized measured data. ΣΔ quantization comparing to conventional pulse code modulation (PCM) is a better quantization choice when we later have to reconstruct the sparse signal from its quantized linearly-measured data. Namely, at each quantization decision step, ΣΔ quantization takes the quantization error arising from the previous quantizing steps into account.
In this talk I will discuss how the linear procedures which are used to get the measured data would affect the error between the real signal and the reconstructed signal.

An SDP Parametric Approach For Solving Single Ratio Quadratic Fractional Programming

Nguyen Van Bong
Department of Mathematics
National Cheng Kung University

This talk focuses on a type of fractional programming which minimizes the ratio of two quadratic functions subject to quadratic constraints taking the following form:

$$\lambda^* = \inf\left\{ \frac{f_1(x)}{f_2(x)} : x \in X \subset \mathbb{R}^n \right\}$$

where $f_i(x) = x^T A_i x + 2b_i^T x + c_i; i = 1, 2$ and $X = \left\{ x \in \mathbb{R}^n : u \leq g(x) \leq v \right\}$ with $g(x) = x^T B x + 2d^T x + \alpha$. It is known that λ^* is the unique root of the following parametric function $f(\lambda)$:

$$f(\lambda) = \inf\left\{ f_1(x) - \lambda f_2(x) : x \in X \subset \mathbb{R}^n \right\}.$$

While the classical Dinkelbach algorithm developed an iterative scheme to approximate λ^*, with the help of S-lemma and some new version of its generation, we show that λ^* can be determined by solving just one SDP under certain conditions. This approach is valid even when $\lambda^* = -\infty$ or λ^* can not be attained in which case the classical Dinkelbach algorithm may fail to apply.

Keywords: Fractional programming, Quadratic programming, SDP relaxation, S-Lemma.

Improved Estimation of Duality Gap in Box Constrained Quadratic Programming Using a Weighted Distance Measure

Ji-Yi Wong
Department of Mathematics
National Cheng Kung University

In this talk, we try to estimate the duality gap between a box constrained nonconvex quadratic programming problem and its Lagrangian dual. The problem takes the following form

$$\text{(P)} \qquad \begin{aligned} &\min f(x) = x^{\mathrm{T}}Qx + 2c^{\mathrm{T}}x \\ &\text{s.t. } x \in [-1,1]^n, \end{aligned}$$

which includes the binary quadratic program as one of its special cases and is thus in general NP-hard. Applying the saddle point theorem, we show that the duality gap estimation can becharacterized as a function of a weighted distance between a certain affine subspace and some parametrized box. Incorporating a technique called the hyperplane arrangement in discrete geometry with different parameters and weights, we now have the flexibility to find a tighter bound for the duality gap. We are also interested in providing a computational way to determine the best bound that can be obtained along the line of approach.

Keywords: quadratic programming, hyperplane arrangement, box constraint, duality gap.