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第二屆台灣邊界元素法與積分方程法研討會

The Second Workshop on Boundary Element Method and Boundary Integral Equation Method in Taiwan

2011 年 11 月 5 日

國家理論科學研究中心(南區) R204

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第二屆台灣邊界元素法與積分方程法研討會
2011 年 11 月 5 日，國家理論科學研究中心(南區)，成功大學

大會議程

日期: 2011 年 **11** 月 **5** 日
地點: 理論中心(南區) R204

時間	演講者	演講題目
09:00 ~ 09:30		**報到，開幕**
09:30 ~ 10:00	呂宗澤	外域問題的基本解方法
10:00 ~ 10:30	楊立杰	機器人側向板的減重與強化
10:30 ~ 11:00		**Tea Break**
11:00 ~ 11:30	陳正宗	邊界元素法中橢圓退化尺度的解析推導與數值實驗
11:30 ~ 12:00	尤玲凰	含有裂縫或支撐的站立薄板的振動
12:00 ~ 13:30		**午餐時間**
13:30 ~ 14:00	夏育群	三維橫向等向性材料熱效應之邊界元素法分析
14:00 ~ 14:30	胡馨云	再生核心質點法求解特徵值問題
14:30 ~ 15:00	黃印良	橢圓方程在有介面不規則區域的數值方法及其誤差估計
15:00 ~ 15:30		**Tea Break**
15:30 ~ 16:00	蔣世中	一種微分積分並列方程式的已知與未知
16:00 ~ 16:30	蔡加正	多重精度用在 Trefftz 法
16:30 ~ 17:00	范佳銘	特雷夫茨法與指數收斂純量同倫法求解反算邊界偵測問題
17:00 ~ 17:20		閉幕
17:30~		Banquet

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2011/11/5, NCTS (South), NCKU

Program

Date: November 5, 2011

Place: CTS (South) R204

時間	演講者	演講題目
09:00 ~ 09:30		**Registration, Opening**
09:30 ~ 10:00	呂宗澤	The Method of Fundamental Solutions for Exterior Problems
10:00 ~ 10:30	楊立杰	Weight Reduction and Reinforcement of Lateral Plate of Rescue Robot
10:30 ~ 11:00		**Tea Break**
11:00 ~ 11:30	陳正宗	Analytical derivation and numerical experiments of degenerate scales for an ellipse in BEM
11:30 ~ 12:00	尤玲凰	Vibration of a Standing Plate with Internal Crack or Support
12:00 ~ 13:30		**Lunch**
13:30 ~ 14:00	夏育群	3D Thermoelastic BEM analysis for Transversely Isotropic Bodies
14:00 ~ 14:30	胡馨云	Reproducing Kernel Particle Method for Elliptic Eigenvalue Problem
14:30 ~ 15:00	黃印良	A Monotone Finite Difference Scheme for Elliptic Interface Problems on Arbitrary Domains
15:00 ~ 15:30		**Tea Break**
15:30 ~ 16:00	蔣世中	Stories of a Class of Integro - Differential Equations
16:00 ~ 16:30	蔡加正	Multiple-precision computing for the Trefftz method
16:30 ~ 17:00	范佳銘	Trefftz method and exponentially convergent scalar homotopy algorithm for the inverse boundary determination problems
17:00 ~ 17:20		Closing
17:30~		Banquet

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外域問題的基本解方法

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摘要

大多數基本解方法的文章處理有界單連通區域上的問題，很少考慮外域的問題。本文使用基本解方法來求解無界區域上的邊值問題。拉普拉斯方程的解在外域的無窮遠點上有兩種條件 (1)$|u| \le c$ 及 (2)$u = O(\ln \rho)$，因此基本解需特別的選取才能滿足此條件。當 $u = O(\ln \rho)$ 時，傳統的基本解可以用。但當 $|u| \le c$ 時，就需要探討新的基本解，我們也簡單地研究此時的誤差分析，數值實驗也驗證了這些理論結果。

基本解方法比其他算法優異之處乃在於其一致的基本解 $\ln r = \ln |PQ|$，其中 P 與 Q 分別是配置點與基底函數的中心點。因此它的算法簡單，程式很容易寫，可省掉很多人力和電腦的計算。故基本解方法相當滿足工程上的需求，節省大量的計算工作與付出。

本文也比較了基本解方法與特解方法的計算結果。此二方法都可視爲相同的 Trefftz 方法，只是使用的基底函數是基本解或特解的不同而已。

我們還研究無界區域上的裂縫問題，用一個結合特解和基本解的 Trefftz 方法來解此問題，其結果與特解方法算出來的一致。因此，本文將基本解方法的應用從有界的單連通區域推廣到更複雜的區域。

關鍵詞：基本解方法、特解方法、邊界近似法、拉普拉斯方程、外域問題、奇異性、誤差分析

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The Method of Fundamental Solutions for Exterior Problems

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Abstract

Most of reports of the method of fundamental solutions (MFS) deal with bounded simply-connected domains, while only a few papers involve in exterior problems. In this talk, we solicit the MFS to seek numerical solutions in unbounded domains. For exterior problems of Laplace's equations, there exist two kinds of infinity conditions, (1) $|u|\leq C$ and (2) $u=O(\ln \rho)$, which must be complied with by the suitable chosen fundamental solutions. For $u=O(\ln \rho)$, the traditional fundamental solutions can be used. However, for $|u|\leq C$, new fundamental solutions are explored, with a brief error analysis. Numerical experiments are carried out to verify the theoretical analysis made.

The remarkable advantage of the MFS over other methods is the uniform FS: $\ln r = \ln |PQ|$, where P and Q are the solution and the source points, respectively. This leads to a simple algorithms and programming, which save a great deal of human power and computer work. Hence the MFS satisfies the engineering requirements by much less computational efforts and little payment.

We also compare the results from the MFS with those by the method of particular solutions (MPS). These two methods can be classified into the Trefftz method (TM) using fundamental solutions (FS) and particular solutions (PS), respectively.

Besides, the crack singularity in unbounded domains is studied as well. A combined TM using both PS and FS is also employed, called combination of the MFS. The numerical results of the MPS and combination of the MFS are coincident with each other. The study in this paper may greatly extend the application of the MFS from bounded simply-connected domains to other complicated domains.

Keywords: method of fundamental solutions, method of particular solutions, Trefftz method, Laplace equation, exterior problem, singularity, error analysis

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機器人側向板的減重與強化

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摘要

如附錄[1]中所敘述，機器人側向板會因爲切去板中某些贅餘的部份，而減輕 1 公斤的質量(由 3 公斤變爲 2 公斤)，但對於其強度影響力卻不大(其允許應力由 —7.1 MPa 降爲 —5.7 MPa)。現在，如果更進一步將其工件材料由鋁合金換爲塑鋼，效果會更爲明顯，但是，其強度卻會因爲材料的更換而大大的降低。除此之外，由於在塑鋼的製作過程中的瑕疵，往往會使機器人側向板受力較大的部份會產生損壞，也因此，這部份的材料將再被鋁合金所取代，本研究的首要目的便是應用邊界元素法，針對這種不同材料的複合機器人側向板，所承受的應力加以分析。由附錄[2]中得知，邊界元素法的第一步，便是沿整個機器人側向板的中心線(介面)，將其分爲兩個部份 B_γ (γ=1, 2)，而對於複合機器人側向板而言，由於材質的不同，會沿介面將其分爲四個部份 B_γ (γ=1, 2, 3 及 4)，如此一來，強化後的機器人，不但可以達到減輕質量的目的，與塑鋼材質的機器人相較，其強度更能因此而大幅的提升。

關鍵詞：鋁合金、塑鋼

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[2] Lih-jier Young and Yeong-pei Tsai, A Boundary Element Application for Mixed Mode Loading Idealized Sawtooth Fracture Surface", Int'l J. of Solids and Structure, 36, pp. 3239-3252, 1999.

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Weight Reduction and Reinforcement of Lateral Plate of Rescue Robot

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ABSTRACT

As described in [1] the weight of one lateral plate of the rescue robot has been reduced 1 kg of mass (from 3 kg to 2 kg) by cutting off some redundancy regions but won't lower the strength that much (allowable stress is reduced from −7.1 MPa to −5.7 MPa) at all. Moreover, the weight of the rescue robot can be reduced more if we replace the aluminum alloy lateral plate by POM. However, the strength of the rescue robot will also be reduced under these circumstances. In addition, because of some defects occur during the manufacturing process failure will happen in the POM lateral plate and therefore, cause the dangerous situation during rescuing process. The heavy loading portion of the POM lateral plate will then be replaced by aluminum alloy to avoid this dangerous situation. The primary purpose of this research is to demonstrate the important of stress analysis of this dissimilar materials compound lateral plate by using the Boundary Element Method (BEM). According to [2] the first step in the BEM solution is to divide the homogeneous medium into two bodies B_γ (γ=1, 2) along the center line which we call the interface. The compound lateral plate will be divided into four bodies B_γ (γ=1, 2, 3 and 4) along the center line of the homogeneous and non-homogeneous medium and will neither heavy as the aluminum one nor easy to failure as the POM. The lightness and rigid robot will be obtained according to the new design of this research.

Keywords: Aluminum Alloy, Polyacetal (POM)

[1] Lih-jier Young, "Numerical Application in Weight Reduction of Lateral Plate of Rescue Robot", Key Engineering Materials, 462-463, pp. 762-767, 2011.
[2] Lih-jier Young and Yeong-pei Tsai, A Boundary Element Application for Mixed Mode Loading Idealized Sawtooth Fracture Surface", Int'l J. of Solids and Structure, 36, pp. 3239-3252, 1999.

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邊界元素法中橢圓退化尺度的解析推導與數值實驗

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[2]台灣，基隆，國立海洋大學，機械與機電工程學系

摘要

本文利用對偶邊界元素法、退化核和單位對數容量來研究橢圓的退化尺度，此退化尺度來源有二：一爲邊界元素法中對數核中積分方程的解不唯一;另一爲複變保角轉換中的保角半徑對應的單位對數容量。退化尺度的數值實驗證據可從執行邊界元素法中看到。而解析方面不僅可從含對數容量保角映射法中來導得退化尺度，而且也可用退化核來導得。藉由使用退化核來導得在弱奇異積分運算子中橢圓領域的特徵值與特徵函數，其中對應的零特徵值即爲導致退化尺度的原因。基於對偶邊界元素法，在弱奇異[U]和強奇異[T]積分算子的左酉向量裡埋藏的假訊息是退化尺度。在另一方面，對強奇異[T]和超強奇異[M]算子的影響係數矩陣作奇異值分解來得到共同的右酉向量之剛體運動訊息。在正常尺度下可解析與數值驗證外域零場和內域非零場。而在齊次 Dirichlet 邊界條件之問題中，退化尺度時可得到相反的內域零場和外域非零場的有趣對應結果。在外域非零場下可確定無失效的 CHEEF 點，固可處處有效來抑制退化尺度的發生。爲了處理非唯一解問題，以力平衡條件來取代以往的剛體想法、CHEEF 和超強奇異邊界積分方程法，可將影響係數矩陣的秩數提升。最後，橢圓的解析和數值的結果將一併探討。

關鍵詞：退化尺度、退化核、對數容量、保角映射法。

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Analytical derivation and numerical experiments of degenerate scales for an ellipse in BEM

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Abstract

Degenerate scale of an ellipse is studied by using the dual boundary element method (BEM), degenerate kernel and unit logarithmic capacity. Degenerate scale stems from either the nonuniqueness of logarithmic kernel in the BIE or the conformal radius of unit logarithmic capacity in the complex variable. Numerical evidence of degenerate scale in BEM is given. Analytical formula for the degenerate scale can be derived not only from the conformal mapping in conjunction with unit logarithmic capacity, but also can be derived by using the degenerate kernel. Eigenvalues and eigenfunctions for the weakly singular integral operator in the elliptical domain are both derived by using the degenerate kernel. It is found that zero eigenvalue results in the degenerate scale. Based on the dual BEM, the rank-deficiency (mathematical) mode due to the degenerate scale is imbedded in the left unitary vector for weakly singular and strongly singular integral operators. On the other hand, we obtain the common right unitary vector of a rigid body (physical) mode in the influence matrices of strongly singular and hypersingular operators after using singular value decomposition. Null fields for the exterior domain and interior nonzero fields are analytically derived and numerically verified in case of the normal scale while the interior null field and nonzero exterior field are obtained for the homogeneous Dirichlet problem in case of the degenerate scale. No failure CHEEF point is confirmed in the nonzero exterior field to overcome the degenerate-scale problem. To deal with the nonuniqueness-solution problem, the constraint of boundary flux equilibrium instead of rigid body term, CHEEF and hypersingular BIEM, is added to promote the rank of influence matrices to be full rank. Both analytical and numerical results agree well in the demonstrative example of an ellipse.

Keywords: degenerate scale, degenerate kernel, logarithmic capacity, conformal mapping.

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in boundary element methods for the two-dimensional Laplace equation, *Engineering Analysis with Boundary Elements*, Vol.26, No.7, pp.559-569, 2002.

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含有裂縫或支撐的站立薄板的振動

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摘要

站立薄板的振動資訊於牆、崁板、窗及加速機動結構體之設計是非常重要的。此振動題目是四階偏微分方程特徵值問題的運用。

在此一演講中，我們將對含有水平裂縫或水平支撐的長方形站立薄板做探討，並且考慮兩垂直邊是支撐邊界，頂邊是自由邊界、支撐邊界，及底邊是固定邊界、支撐邊界之情況。我們將用一種半解析數值方法–Levy 積分法和一種接合方法計算其基礎振動頻率及振動模態。裂縫、支撐、薄板之形狀、薄板之重量及薄板邊界條件對基礎振動頻率之影響將被探討。

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Vibration of a Standing Plate with Internal Crack or Support

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Abstract

Vibration of a standing plate under self-weight is important in the design of walls, panels, windows, and accelerating mobile structures. It is related to an eigenvalue problem with a 4th order partial differential equation.

In this talk, a standing rectangular plate with horizontal internal crack or support, having simply supported vertical edges, free or simply supported top edge, and simply supported or clamped bottom edge, will be considered. A semi-analytic Levy-integration method and a matching method will be used to compute the fundamental frequency and the mode shape of the vibration of the plates. The effect of the crack, the support, the shape of the plate, the weight of the plate, and the edge conditions on the vibration of the plate will be investigated.

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三維橫向等向性材料熱效應之邊界元素法分析

夏育群
逢甲大學航太與系統工程系

摘要

利用邊界元素法分析三維異向彈性力學並且考慮熱效應對材料之問題時，此時在其邊界積分方程式(Boundary Integral Equation)會產生一體積分項，任何數值積分法皆會破壞邊界求解法之概念。然而，在過去十年內，三維異向熱彈之邊界積分式轉換方法還尚未成功。主要困難的原因爲三維異向體基本解之數學式非常複雜。爲了進一步突破解決一般三維全異向性熱彈問題，本研究論文是針對橫向等向性材料來探討分析，藉由邊界元素法理論，分析材料受力並且有熱效應時，探討所產生的位移與應力，並撰寫成 FORTRAN 程式碼，只要依規定之格式輸入所需要的資料，就可以計算出其邊界之位移和應力。最後提出幾個實際範例，與有限元素法套裝軟體 ANSYS 分析結果進行比對，來驗證比較其結果之準確性和誤差值。

關鍵字：橫向等向性熱彈分析、邊界元素法、體積分轉換面積分

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3D Thermoelastic BEM analysis for Transversely Isotropic Bodies

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Abstract

When analyzing 3D generally anisotropic thermoelasticity problem in conjunction with the material thermal effect, the boundary integral equation will generate a volume integral term. Any direct numerical integration of the extra volume integral shall require domain discretization that will destroy the BEM's notion as a truly boundary solution technique. However, such integral transformation for 3D anisotropic thermoelasticity has not been accomplished yet in the past decades. The main difficulty stems from the mathematical complexity of the fundamental solutions for 3D anisotropic bodies. As the proscenium to further treat 3D generally anisotropic thermoelasticity, this paper focuses on making the volume-to-surface integral transformation (VIT) for 3D transversely isotropic bodies. Inputting necessary data according to the fixed form of the program developed by FORTRAN codes, one may then calculate the elastic field on the boundary. At last, a few numerical examples are presented to demonstrate the veracity and accuracy by comparing the results with Ansys.

Keywords: Transversely isotropic thermoelasticity, Boundary element method, Volume-to-surface integral transformation

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再生核心質點法求解特徵值問題

胡馨云，吳育璇，潘金龍，陳俊賢
東海大學數學系

摘要

無網格法大致上可分爲兩類，一類是由弱型式衍生出來的，而另一類是由強型式衍生而出。此論文使用弱型式中的再生核心質點法求解特徵值問題，著重於收斂性分析，此法的基底函數是局部基底，所以離散後的特徵系統是稀疏帶狀的，相似於有限元素法的結構。其帶寬較有限元素法的寬一些，收斂情形比有限元素法好。從理論分析上得知，特徵函數誤差的收斂率與再生核心基底函數的階及半徑有關，特徵值的誤差與特徵函數在一範數下的誤差成平方關係。在數值結果與理論相吻合，可知選用二階，p=2，效果最好，當然可選再高階，例如 p=3，但相對計算時間需更多、更具彈性。在國際間，再生核心質點法被視爲廣泛型有限元素法。

The Second Workshop on Boundary Element Method and Boundary Integral Equation Method
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Reproducing Kernel Particle Method for Elliptic Eigenvalue Problem

H. Y. Hu, Y. H. Wu, J. L. Pan & J. S. Chen

Abstract

The development of meshfree methods can be traced back from two branches, one based on weak form and other based on strong form. In this work, we use the reproducing kernel particle method (RKPM) for solving eigenvalue problem under weak form. We concentrate on the convergence analysis; the basis function of this method is local. So the eigen system is sparse and band after discrete, similarly as finite element method. However the band width of RKPM is more widely than finite element method and the convergence of RKPM is better than finite element method. A result extracted from the analysis is that the convergence about the errors of eigen functions is related to the degrees and radius of RKPM. The errors of eigenvalues are present square relations to the errors of eigen functions under one norm. Some numerical experiments provided to validate the results of error analysis, we found out that the best effect when we chose the degree of two, p=2. We could chose higher of degree, for example p=3, but its waste much time to compute. RKPM is more flexible than finite element method, as a whole and it can be viewed as a generalized finite element method.

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橢圓方程在有介面不規則區域的數值方法及其誤差估計

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摘要

我們利用與介面重合的曲線座標提出基於有限差分法的數值方法來求解橢圓介面問題。離散之後得到的線性系統是對稱且定性的，所以可以利用諸如 PCG 或是多重網格等有效的方法來反解。又因爲我們的數值方法保持算子的單調性，滿足極大值原理，所以可以嚴格地證明數值誤差的最大模具有二階精度。另外，此數值方法亦可求出跨過介面的法向流至二階精度。針對介面上的不連續條件的處理，自然地包含在差分格式中，不需要做額外的處理，所以易於編程。解在介面上的不連續性可以被刻畫出，不會因爲數值誤差而被光滑化。數值求解的結果進一步確認理論估計的誤差的確到二階精度。

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A Monotone Finite Difference Scheme for Elliptic Interface Problems on Arbitrary Domains

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Abstract

We use body-fitting curvilinear coordinates to construct a finite difference scheme for the elliptic interface problems. The resultant matrix is symmetric and definite, hence can be inverted efficiently by powerful linear solvers such as PCG and multigrid. Because the scheme is elaborated to preserve the monotonicity, the global errors are proved to achieve second-order accuracy in the sup-norm. In addition, the scheme can reconstruct the numerical flux at the interface with a second-order convergence rate. The jump conditions are automatically captured in the scheme without extra enforcement. Thus the approximation keeps the discontinuities at the interface without numerical smearing at all. Extensive numerical experiments are performed to demonstrate and confirm our theoretical results.

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一種微分積分並列方程式的已知與未知

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摘要

第一部份，我們先介紹一種由空氣動力學中所衍生出的特殊微分積分並列方程式。由於物理意義，原始方程式是定義在無限區間的，但這在數學的討論上是有困難的；尤其是數值方面。所以我們也討論有限區間版本，並介紹兩種版本之間的關係。接下來，我們將討論有限版本的解析解與數值解的求法及所得結果。

第二部份，我們將第一部份的方程式改變爲最佳控制形式，再討論其最佳表現、最佳形態及最佳控制等問題。

第三部份，我們將提出幾個潛在性、尚未明朗的問題作爲結束。

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Stories of a Class of Integro - Differential Equations

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Abstract

In this talk, we would like to introduce a class of integro - differential equations. This kind of equations was originally from describing high speed airfoil motion in an incompressible flow. There are two parts in this system: one part contains integro - differential Volterra equations, the other part is a singular integro - differential equations. We will discuss the infinite version, finite version and their relationship of the singular integro - differential equations. In this kind of equations, we are able to find the unique exact solutions, and we do it in both ways: analytically and numerically.

In the second part, we will discuss the associated issues with optimal controls: the performance evaluations, the ideas and the ways to find the optimal controls and states.

In the third part, we would like to propose some open questions for future research.

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多重精度計算用在 Trefftz 法

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摘要

Trefftz method 是指數收斂的數值方法，換言之，對數的誤差正比於空間離散化中的節點數目。而本研究中用 Trefftz method 來求解 Laplace 方程式。在解題的過程中，Trefftz 法的不穩性的系統矩陣，可以使用 MPFR (multiple precision floating-point reliable)來避免。初步結果顯示，在面積/周長比率較大的時，收斂速度會更快。同時我們也發現，我們的方法沒有尺度問題，換句話說，增加離散點數所需的時間，只有代數成長。

關鍵詞：指數收斂、**Trefftz** 法、多重精度庫

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Multiple-precision computing for the Trefftz method

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Abstract

The Trefftz method is a numerical method of exponential convergence. In other word, the logarithmic error is proportional to the node number of spatial discretization. In this research, the exponential convergence of Trefftz method is demonstrated by solving Laplace equation in domains of ellipses and amoeba shapes. In the solution procedure, the ill-conditioning of system matrix of the Trefftz method can be avoided by using the multiple precision floating-point reliable (MPFR) library. The results converge faster for the cases of smoother boundary conditions and larger area/Circumference ratio. In addition, we also found the computation is scalable in the sense that the required time increases only algebraically.

Keywords: exponential convergence, Trefftz method, multiple precision floating-point reliable library

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特雷夫茨法與指數收斂純量同倫法求解反算邊界偵測問題

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摘要

在本研究中，我們提出使用特雷夫茨法來分析邊界偵測問題。所求解邊界偵測問題之控制方程式包含二維拉普拉斯方程式、赫姆霍茲方程式、修正型赫姆霍茲方程式以及雙調和方程式。在邊界偵測問題中，一部分邊界的空間位置屬於未知狀態，因此邊界偵測問題也是反算問題的一種。由於一部分邊界的空間位置未知，因此在已知位置的邊界上需給予較多的邊界條件，也就是柯西邊界條件。特雷夫茨法是屬於一種邊界配點型態的無網格法，具有較高的計算效率與準確性。在特雷夫茨法中，數值解能以完備基底之線性累加所表示。無網格法相較於傳統的數值方法，在空間離散時不需要進行網格產生和數值積分，且數值計算操作也較爲簡單，可以減少所需之電腦資源與增加計算效率。採用邊界型態配點的無網格法來進行空間離散，可以避免重覆重建網格。因爲邊界的位置未知，在進行空間離散時，在計算域上所配置的每一個邊界點，都會產生非線性代數方程式，最後會形成一組非線性代數方程式組。

本文採用指數收斂純量同倫法來求解非線性代數方程式組。指數收斂純量同倫法引入虛擬時間以及純量同倫函數之概念，能夠求解過定或欠定系統，並具有全域收斂及指數收斂之特性，在演化過程中對於初始猜值較不敏感，故本研究採用此方法來求解非線性代數方程式組。在本文中會以數個數值算例來驗證所提出數值方法的可行性及準確性。在實際問題中，人爲或機器因素會造成邊界值量測誤差之產生，因此需要證明數值模擬方法之穩定性與抗噪性，在數值算例中所給定之邊界條件上，給予噪音之擾動，藉此測試數值方法的穩定度，並證明本文所提出方法的可行性與效率性。

關鍵詞：無網格法，特雷夫茨法，拉普拉斯方程式，赫姆霍茲方程式，修正型赫姆霍茲方程式，雙調和方程式，邊界偵測問題，指數收斂純量同倫法。

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Trefftz method and exponentially convergent scalar homotopy algorithm for the inverse boundary determination problems

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Abstract

In this study, the combination of the Trefftz method and the exponentially convergent scalar homotopy algorithm (ECSHA) are proposed for solving boundary determination problems governed by Laplace equation, Helmholtz equation, modified Helmholtz equation and biharmonic equation. In the boundary determination problem, the spatial position for part of boundary was unknown in *a priori*. One kind of boundary conditions is given on this portion of boundary with unknown spatial position. One the other hand, over-specified boundary conditions are imposed on the boundary portion with known spatial position. Therefore, the boundary determination problem is one kind of inverse problems. In order to stably and efficiently solve the boundary determination problem, we proposed a boundary-type meshless method which is based on the Trefftz method and the ECSHA. By adopting the Trefftz method, which is meshless and integral-free, the numerical solutions are expressed by a linear combinations of the T-complete functions of the governing equation. When the satisfactions of the boundary conditions are imposed on the boundary points, it will result in a system of nonlinear algebraic equations (NAEs).

In this study, the ECSHA is adopted to efficiently obtain the convergent solution of the system of NAEs. In ECSHA, the concepts of fictitious time and scalar homotopy function are introduced to derive the evolutionary equation. Besides, it can be used to solve the over-determined system or under-determined system. Furthermore, the ECSHA is insensitive to initial guess of the evolutionary process. The efficiency of computation can be greatly improved since the calculation for inverse of Jacobian matrix can be avoided in comparing with the well-known Newton's method.

Finally, there are some numerical examples provided to validate the ability and accuracy of

the proposed scheme. Besides, some factors that might influence on the performance of the proposed meshless scheme will be examined through a series of numerical experiments. The stability of the proposed scheme can be verified by adding some noise to the boundary conditions.

Keywords: meshless method, Trefftz method, Laplace equation, Helmholtz equation, modified Helmholtz equation, biharmonic equation, boundary determination problems, exponentially convergent scalar homotopy algorithm.


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