Complex symmetric stabilizing solution of the nonlinear matrix equation $X + A^{\top}X^{-1}A = Q$

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Abstract

We study the matrix equation $X + A^{\top}X^{-1}A = Q$, where A is a complex square matrix and Q is complex symmetric. Special cases of this equation appear in Green's function calculation in nano research and also in the vibration analysis of fast trains. In those applications, the existence of a unique complex symmetric stabilizing solution has been proved using advanced results on linear operators. The stabilizing solution is the solution of practical interest. In this talk we will give an elementary proof of the existence for the general matrix equation, under an assumption that is satisfied for the two special applications. Moreover, our new approach here reveals that the unique complex symmetric stabilizing solution has a positive definite imaginary part. The unique stabilizing solution can be computed efficiently by the doubling algorithm.

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