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Matrix with Spectrum Outside its k-th Order Geršgorin's Region

Abstract

Let $A = (a_{ij})_{n\times n}$ be a complex matrix and

$$R_i = \sum_{k=1,k\neq i}^{n} |a_{ik}| \quad for i = 1, \cdots, n.$$

Its k-th order Geršgorin region is defined by

$$G_k(A) = \bigcup_{1\leq \rho_1<\cdots<\rho_k\leq n} \{z \in \mathbf{C} : \prod_{i=1}^{k} |z - a_{\rho_i\rho_i}| \leq \prod_{i=1}^{k} R_{\rho_i}\}.$$

In view of the Geršgorin Disk Theorem, it is natural to ask its generalization if the spectrum $\sigma(A) \subset G_k(A)$ for $k \leq n$. It turns out to be false with some counter-examples. Then Newman and Thompson in 1994 raised the open question: For what value of n and k, $3 \leq k \leq n$, there exists an $n \times n$ mtrix A with $G_k(A)\cap\sigma(A) = \phi$. This paper is going to answer this problem: such matrices exist for all pairs of n and k.