Genuine High-Order Einstein-Podolsky-Rosen Steering

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Einstein-Podolsky-Rosen (EPR) steering demonstrates that two parties share entanglement even if the measurement devices of one party are untrusted. Here, going beyond this bipartite concept, we develop a novel formalism to explore a large class of EPR steering from generic multipartite quantum systems of arbitrarily high dimensionality and degrees of freedom, such as graph states and hyperentangled systems. All of these quantum characteristics of genuine high-order EPR steering can be efficiently certified with few measurement settings in experiments. We faithfully demonstrate for the first time such generality by experimentally showing genuine four-partite EPR steering and applications to universal one-way quantum computing. Our formalism provides a new insight into the intermediate type of genuine multipartite Bell nonlocality and potential applications to quantum information tasks and experiments in the presence of untrusted measurement devices.

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Einstein-Podolsky-Rosen (EPR) steering was originally introduced by Schrödinger in 1935 [1] to describe the EPR paradox [2]. Recently, it has been formulated by Wiseman *et al.* [3] to show a strict hierarchy among Bell nonlocality, steering, and entanglement [4,5] and stimulated new applications to quantum communication [6]. Several experimental demonstrations of EPR steering have been reported [5,7,8]. The steering effect reveals that different ensembles of quantum states can be remotely prepared by measuring one particle of an entangled pair. We go a step further and consider the following question: how to experimentally observe genuine multipartite EPR steering? For instance, given an experimental output state ρ_{expt} which is created according a target four-qubit cluster state of the form [9,10]

$$|G_4\rangle = \frac{1}{2}(|+\rangle_1|0\rangle_2|+\rangle_3|0\rangle_4 + |-\rangle_1|1\rangle_2|+\rangle_3|1\rangle_4 + |+\rangle_1|0\rangle_2|-\rangle_3|1\rangle_4 + |-\rangle_1|1\rangle_2|-\rangle_3|0\rangle_4),$$

where $|\pm\rangle_k = (|0\rangle_k \pm |1\rangle_k)/\sqrt{2}$, how do we describe the effect of genuine multipartite EPR steering and then detect such steerability of ρ_{expt} in the laboratory?

Inspired by the task-oriented formulation of bipartite steering [3], genuine multipartite EPR steering can be defined from an operational interpretation as the distribution of multipartite entanglement by *uncharacterized* (or untrusted) parties. Let us consider a system composed of N parties and a source creating N particles. Each party of the system can receive a particle from the source whenever an N-particle state is created. We divide the system into two groups, say A_s and B_s, and assume that A_s is responsible for sending particles from such a source to every party. Each time, after receiving particles, they measure their respective parts and communicate classically. Since B_s does not trust A_s, A_s's task is to convince B_s that the state shared between them is entangled. A_s will succeed in this task if and only if A_s can prepare different ensembles of quantum states for B_s by steering B_s's state. Here we say an N-particle state generated from the source to possess genuine N-partite EPR steerability if by which A_s succeed in the task for *all* possible bipartitions A_s and B_s of the N-particle system. This interpretation is consistent with the definition recently introduced by He and Reid [11]. In a wider scope, Schrödinger's original concept can even be applied to quantum systems with many degrees of freedom (DOFs), e.g., hyperentangled systems, and, as will be shown presently, extended as *genuine* multi-DOF EPR steering. In this Letter we call these two sorts of quantum characteristics genuine high-order EPR steering.

The concept of verifying genuine high-order EPR steering leads us naturally to consider quantum scenarios based on genuine high-order entanglement [12–17] in which B_s's measurement apparatus are trusted, while A_s's are not; see Fig. 1. Demonstrations of genuine high-order EPR steerability guarantee faithful implementations of the quantum scenarios in the presence of uncharacterized parties. So far, while verifying genuine tripartite steering becomes possible [11], the fundamental problem such as the verification considered above and the cases for arbitrary large N remains open.

Here we develop new *quantum witnesses* to observe a large class of genuine high-order EPR steering independent

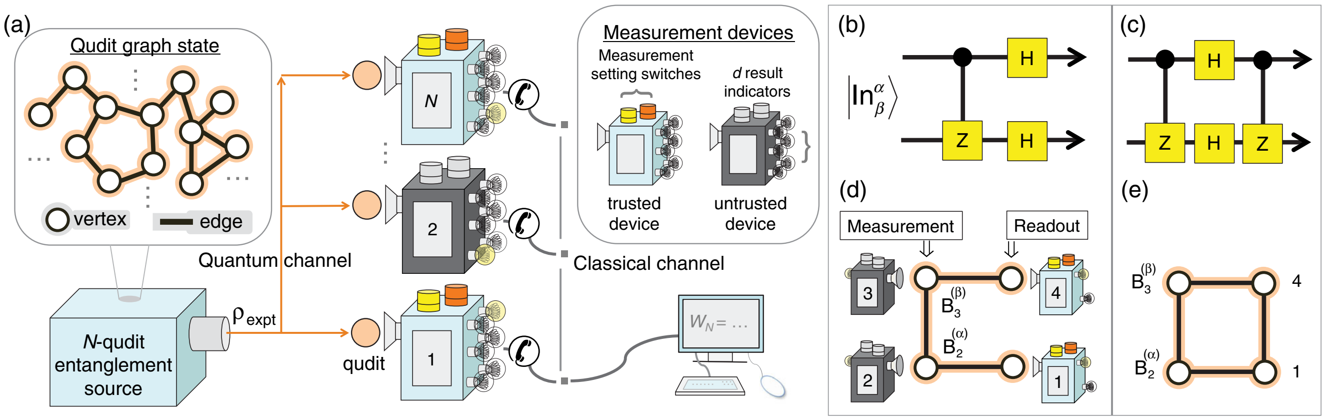

FIG. 1 (color online). Genuine multipartite EPR steerability and applications. Generic entanglement-based quantum protocols rely on both characterized measurement devices and genuine multipartite entanglement, such as one-way quantum computation [12,13] and multiparty quantum communications [14]. Genuine multipartite EPR steerability enables them to perform quantum applications in the presence of untrusted measurement apparatus (a). Here, we develop quantum steering witnesses to ensure that an experimental state $\rho_{\rm expt}$ of an N-partite quantum d-dimensional system (N qudits) has such ability, for example, an experimental graph state [18–21] used for one-way computing. To implement gate operations, such as circuits, composed of one-qubit ($d=2$) gate $\hat{H}$ and two-qubit controlled-Z (CZ) gates (b) and (c) for input states $|{\rm In}^{\alpha}_{\beta}\rangle$, one needs to prepare chain-type (d), i.e., $|G_4\rangle$, and box-type (e) cluster states, respectively. By performing measurements $B_2^{(\alpha)}$ and $B_3^{(\beta)}$ on qubits 2 and 3, respectively, the rest of the qubits 1,4 together with the outcomes of measurements on qubits 2, 3 would provide a readout of the gate operations. See the Supplemental Material for detailed discussions [21]. In addition to releasing the assumptions about the measurement devices, genuine multipartite steerability promises high-quality one-way computation (see Fig. 3).

of the particle or DOF number. Let us start with a demonstration of the first experimental genuine four-partite EPR steerability for states close to the cluster state $|G_4\rangle$. In our scenario, we assume that two possible measurements can be performed on each particle ($m_k = 1,2$ for the kth particle) and that each local measurement has two possible outcomes, $v_k^{(m_k)} \in \{0,1\}$. We take the measurements for each party who implements quantum measurements to observables with the nondegenerate eigenvectors $\{|0\rangle_{k,1}=|0\rangle_k, |1\rangle_{k,1}=|1\rangle_k\}$ for $m_k = 1$ and $\{|0\rangle_{k,2} = |+\rangle_k, |1\rangle_{k,2} = |-\rangle_k\}$ for $m_k = 2$.

The genuine four-partite EPR steerability of the ideal cluster state $|G_4\rangle$ is revealed by the following relations of measurement results: $v_1^{(1)}+v_2^{(2)}+v_3^{(1)}\doteq 0$ and $v_3^{(1)}+v_4^{(2)}\doteq 0$, and the relation: $v_2^{(1)}+v_3^{(2)}+v_4^{(1)}\doteq 0$ and $v_1^{(2)}+v_2^{(1)}\doteq 0$, where $\doteq$ denotes equality modulo 2. When A_s and B_s share a state $|G_4\rangle$, A_s can steer the states of B_s's particles by measuring the particles held as described by the above relations whatever the partition A_s and B_s is considered. Thus, we construct the kernel of quantum steering witness in the form

$$W_4 \equiv P(v_1^{(1)}+v_2^{(2)}+v_3^{(1)}\doteq 0, v_3^{(1)}+v_4^{(2)}\doteq 0) + P(v_2^{(1)}+v_3^{(2)}+v_4^{(1)}\doteq 0, v_1^{(2)}+v_2^{(1)}\doteq 0),$$

where $P(\cdot)$ denotes the joint probability of obtaining experimental outcomes satisfying the designed conditions. If B_s has a preexisting state known to A_s, rather than part of a genuine multipartite entanglement shared with A_s, the maximum value of W_4 is

$$W_{4C} \equiv \max_{\mathbf{A}_s,\{\mathbf{v}_a\}_C,\{\mathbf{v}_b\}_{\rm QM}} W_4 = 1+\frac{1}{\sqrt{2}} \sim 1.7071,$$

where $\mathbf{A}_s$ denotes the index set of A_s for all possible partitions and $\{\mathbf{v}_a\}_C$ indicates that the outcome set for A_s is derived from such a preexisting-state scenario. The outcome set $\{\mathbf{v}_b\}_{\rm QM}$ of B_s is obtained by performing quantum measurements on the preexisting quantum states. Hence, we posit that if an experimental state $\rho_{\rm expt}$ shows that

$$W_4(\rho_{\rm expt}) > 1+\frac{1}{\sqrt{2}}, \qquad (1)$$

then $\rho_{\rm expt}$ can exhibit genuine four-partite EPR steerability close to $|G_4\rangle$. This certification rules out all the possibilities of results mimicked by tripartite steerability, including all possible mixtures of them. States certified by this witness will enable the quantum protocols to be implemented even when uncharacterized measurement apparatus are unavoidably used (Fig. 1). It is also worth noting that the steering witness (1) is efficient. Only the *minimum* two local measurement settings are sufficient to show the steerability.

To experimentally observe the genuine-four partite EPR steerability, we utilize the technique developed in the previous experiment [22] to generate a source of two-photon four-qubit cluster states entangled both in polarization and spatial modes. As illustrated in Fig. 2, an ultraviolet (UV) pulse passes twice through two contiguous type-I–β barium borate (BBO) to produce polarization entangled photon pairs in the forward (spatial modes $R_{A,B}$) and the backward ($L_{A,B}$) directions. Through

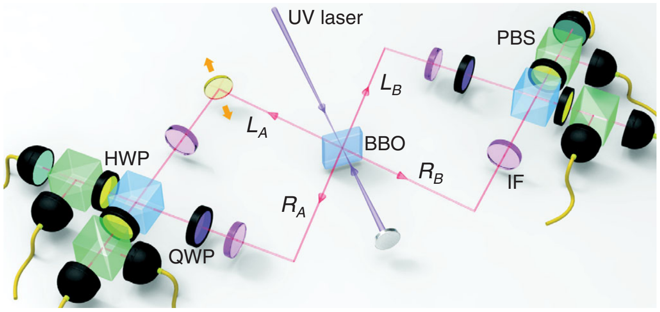

FIG. 2 (color online). Experimental setup. A pulse (5 ps) of UV light with a central wavelength of 355 nm and an average power of 200 mW at repetition rate of 80 MHz double passes a two-crystal structured BBO to produce polarization entangled photon pairs either in the forward direction or in the backward direction. To create desired entangled pairs in mode R_A, R_B and in L_A, L_B, two quarter wave plates (QWPs) are properly tilted along their optic axis. Half wave plates (HWPs), polarizing beam splitters (PBSs), and eight single-photon detectors are used as polarization analyzers for the output states. Here 3-nm bandpass filters (IFs) with central wavelength 710 nm are placed in front of them.

temporal overlaps of modes R_A and L_A and of modes R_B and L_B, one can create a four-qubit state

$$|G_4'\rangle = \frac{1}{2}[(|H\rangle_A|H\rangle_B + |V\rangle_A|V\rangle_B)|R\rangle_A|R\rangle_B + (|H\rangle_A|H\rangle_B - |V\rangle_A|V\rangle_B)|L\rangle_A|L\rangle_B], \quad (2)$$

entangled both in spatial modes and horizontal (H) and vertical (V) polarizations. By encoding logical qubits as $|H(V)\rangle_{A(B)} \equiv |0(1)\rangle_{1(2)}$ and $|R(L)\rangle_{A(B)} \equiv |0(1)\rangle_{3(4)}$, $|G_4'\rangle$ is equivalent to the cluster state $|G_4\rangle$ up to a transformation $\hat{H}_1 \otimes \hat{H}_4$, where $\hat{H}_k|0(1)\rangle_k = |+(-)\rangle_k$. The witness kernel for this target state has a corresponding change in measurement settings by $W_4' \equiv P(v_1^{(2)} + v_2^{(2)} + v_3^{(1)} \doteq 0, v_3^{(1)} + v_4^{(1)} \doteq 0) + P(v_2^{(1)} + v_3^{(2)} + v_4^{(2)} \doteq 0, v_1^{(1)} + v_2^{(1)} \doteq 0)$.

In the experiment, we obtain a high generation rate of cluster state about 1.2×10^4 per sec with 200 mW UV pump. We measure the two joint probabilities in the witness kernel with the designed measurement settings [23] and have $P(v_1^{(2)} + v_2^{(2)} + v_3^{(1)} \doteq 0, v_3^{(1)} + v_4^{(1)} \doteq 0) = 0.9490 \pm 0.0022$ and $P(v_2^{(1)} + v_3^{(2)} + v_4^{(2)} \doteq 0, v_1^{(1)} + v_2^{(1)} \doteq 0) = 0.9339 \pm 0.0027$. Then we observe genuine four-partite steerability verified by

$$W_4'(\rho_{\text{expt}}) = 1.8829 \pm 0.0049. \quad (3)$$

This result is clearly larger than the maximum value the preexisting-state scenario can achieve, W_{4C}, by ~36 standard deviations.

The measured witness kernel $W_4(\rho_{\text{expt}})$ can also reveal the information about the state fidelity, $F_S(\rho_{\text{expt}}) = \text{Tr}[|G_4\rangle\langle G_4|\rho_{\text{expt}}]$. To derive such connection, let us first represent the witness (1) in the operator form, $\hat{W}_{G_4} \equiv W_{4C}\hat{I} - \hat{\mathcal{W}}_4$, where $\hat{I}$ denotes the identity operator and $\hat{\mathcal{W}}_4$ corresponds to W_4 such that $W_4(\rho_{\text{expt}}) = \text{Tr}[\hat{\mathcal{W}}_4\rho_{\text{expt}}]$. If $\langle \hat{W}_{G_4}\rangle = \text{Tr}[\hat{W}_{G_4}\rho_{\text{expt}}] < 0$, then ρ_{expt} is genuinely four-partite steerable. One can use $\hat{W}_{G_4}$ to derive a steering witness operator of the form $\hat{W}'_{G_4} \equiv W_{4C}/2\hat{I} - |G_4\rangle\langle G_4|$ [21], which provides a steering witness $F_S(\rho_{\text{expt}}) > W_{4C}/2$ (~0.8536). They satisfy the relation $\hat{W}'_{G_4} - \gamma\hat{W}_{G_4} \geq 0$, where γ is some positive constant, which means that when a state is detected by $\hat{W}'_{G_4}$, it is certified by $\hat{W}_{G_4}$ as well. Such a relation can be manipulated to give $|G_4\rangle\langle G_4| \leq \hat{\mathcal{W}}_4/2$, i.e., to provide the upper bound of the state fidelity, $F_S(\rho_{\text{expt}}) \leq W_4(\rho_{\text{expt}})/2$. Similarly, we use $\hat{W}'_{G_4}$ to construct another steering witness operator composed of $\hat{\mathcal{W}}_4$ [21] and derive the lower bound of $F_S(\rho_{\text{expt}})$: $\hat{\mathcal{W}}_4 - \hat{I} \leq |G_4\rangle\langle G_4|$. Hence, $F_S(\rho_{\text{expt}})$ is estimated as

$$0.8829 \pm 0.0049 \leq F_S(\rho_{\text{expt}}) \leq 0.9415 \pm 0.0025. \quad (4)$$

Such characteristic of genuine four-partite steering serves as a source for implementing faithful one-way quantum computing. We have realized the quantum gates illustrated in Fig. 1 [21] and objectively evaluated their performance and connection with steerability. We use $W_4'(\rho_{\text{expt}})$ Eq. (3) to estimate three different fidelities [24]: their average computation fidelity (F_{comp}) [25], the quantum process fidelity (F_{process}) [26], and the average state fidelity (F_{av}) [26]; see Fig. 3. To connect steerability with the gate operation shown in Fig. 1(b), we first construct a steering witness operator of the form $\hat{W}_{\text{CZ}} \equiv W_{4C}\hat{I} - \hat{\mathcal{W}}_{\text{CZ}}$ [21], where the operator $\hat{\mathcal{W}}_{\text{CZ}}$ specifies the relation between ideal input and output states of the quantum circuit [27]. This means the steerability can be verified by performing one-way computation to check $\langle\hat{\mathcal{W}}_{\text{CZ}}\rangle > W_{4C}$. Following the same method as that used to estimate the lower and upper bounds of $F_S(\rho_{\text{expt}})$ [21], we get $W_4(\rho_{\text{expt}}) - 1 \leq \langle\hat{\mathcal{W}}_{\text{CZ}}\rangle \leq W_4(\rho_{\text{expt}})/2 + 1$. Hence, with the relation $F_{\text{comp}} = \langle\hat{\mathcal{W}}_{\text{CZ}}\rangle/2$ [27] and the estimation of $\langle\hat{\mathcal{W}}_{\text{CZ}}\rangle$, we arrive at the steering witness $F_{\text{comp}} > W_{4C}/2$ and the fidelity estimation

$$W_4(\rho_{\text{expt}}) - 1 \leq F_{\text{comp}} \leq \frac{1}{4}W_4(\rho_{\text{expt}}) + \frac{1}{2}. \quad (5)$$

F_{process} and F_{av} are further determined by F_{comp} [24].

The concept and method of the criterion (1) can be directly applied to quantum states with complex structures. One of the extensions is to certify steerability of a general d-dimensional N-partite and q-colorable graph state $|G\rangle$ [18–20] [Fig. 1(a)]. The kernel of the steering witness is of the form

$$W_N(q,d) \equiv \sum_{m=1}^{q} P\left(v_j^{(2)} + \sum_{i\in\epsilon(j)} v_i^{(1)} \doteq 0 | \forall j \in Y_m\right),$$

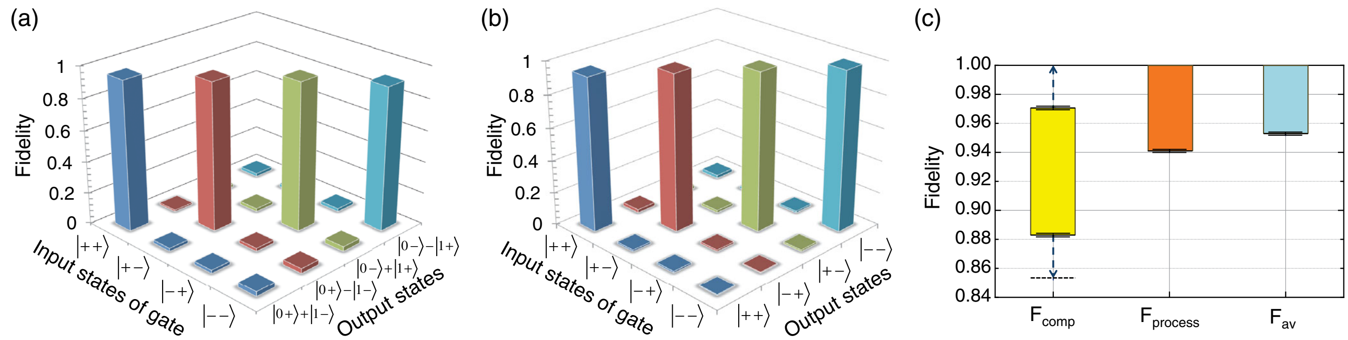

FIG. 3 (color online). Steerability and performance of quantum gates. Two kinds of gate operations are experimentally realized for the target quantum gates [Figs. 1(b), 1(c)]. We measure the fidelities of the output states by inputting four orthogonal states into the experimental gates, $|mn\rangle$ for $m, n = +, -$ [24]: (a) a mean fidelity $\sim 0.935 \pm 0.004$ for the gate operation, Fig. 1(b), and (b) an average fidelity $\sim 0.960 \pm 0.004$ for the target gate, Fig. 1(c). (c) From the estimate of the average computation fidelity F_{comp} (5), we obtain the lower bounds of the quantum process fidelity F_{process} ($\sim 0.9415 \pm 0.0025$) and the average state fidelity F_{av} ($\sim 0.9532 \pm 0.0020$), which indicate good qualities of our experimental gates, regardless of the input states [24]. Since both created gates are based on the same source, their estimations of the three different fidelities are identical [21]. The gate performance reveals genuine four-partite steerability by the steering witness $F_{\text{comp}} > W_{4C}/2 \sim 0.8536$. The double-arrow dashed line shows the region where the source states are truly four-partite steerable.

for $v_i^{(1)}, v_j^{(2)} \in \{0, 1, \ldots, d-1\}$ measured in two complementary bases [21], where $\doteq$ denotes equality modulo d and $\epsilon(j)$ represents the set of vertices that form edges with the vertex j in the color set Y_m. The quantum steering witness is described by

$$W_N(q, d|\rho_{\text{exp}}) > \frac{1}{2}\left(q + \sqrt{\frac{2(q^2 - 2q + 2) + \gamma_q}{d}}\right), \qquad (6)$$

where $\gamma_2 = 0$ and $\gamma_q = [2(q-3)+1] + \gamma_{q-1}$ for $q \geq 3$ [21]. If ρ_{expt} is detected by Eq. (6), then ρ_{expt} possesses genuine N-partite EPR steerability close to $|G\rangle$. This witness has the same features as the witness (1) [28] and possesses high robustness against noise [21]. With only q local measurement settings, $W_N(q, d)$ can be efficiently realized regardless of the number of qudits. We remark that, for states that do not belong to the above state types, for example, the W states [29], useful steering witnesses still could be derived from one's knowledge to this target state [21]. For more extensions, how to observe EPR steering in *all* DOFs [30,31] under consideration is shown in Supplemental Material [21]. The concrete experimental illustrations and applications of such genuine high-order EPR steering are also detailed therein [21].

In conclusion, we have developed a novel formalism to explore genuine high-order EPR steering and experimentally demonstrated such generality and applications with photonic cluster states. Being capable of revealing genuine high-order EPR steering pushes beyond the capability of bipartite steering and promotes potential applications and experiments. One can probe more sorts of steerability, for example, in resonating valence-bond states, which would allow an analogue quantum simulator [15] to run without fully characterized quantum measurements. Similarly, other quantum strategies based on both characterized measurements and genuine multipartite entanglement, like quantum metrology [16], can benefit from it as well. Moreover, since genuine multipartite EPR steerability cannot be mimicked by parts of the whole system, such ability together with steering witness could facilitate multipartite secret sharing [14] in a generic one-sided device-independent mode [6,11]. It is interesting to compare our method with the recently developed single-system steering for quantum information processing [32] and to investigate further from the all-versus-nothing (AVN) point of view. Subtle AVN proof for steering and their experimental demonstration are given for special classes of two qubits [33,34].

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Note added.—Recently, we became aware of two experimental demonstrations of tripartite steering [35,36].

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[23] We use three different setups to implement spatial mode analysis. When performing the measurement $m_k = 2$ for $k = 3$ (4), a beam splitter is set at the intersection of modes $R_{A(B)}$ and $L_{A(B)}$. Furthermore, combined with α-phase shifter, measurement $B_k^{(\alpha)}$ can be realized for one-way quantum computing [Figs. 1(d), 1(e) and [21]]. Whereas for the measurement $m_k = 1$, they are left empty. If a PBS is placed at the mode intersection, one can realize the measurements $m_{1,2} = 1$ and $m_{3,4} = 1$, simultaneously.

[24] Here, F_{comp} is considered by using two complementary sets of input states [25]. F_{process} determines the similarity between the ideal and experimental process of quantum gates [26]. F_{av} is the quantum state fidelity between the ideal and experimental gate outputs uniformly averaged over all input states on a state space [26]. There exist useful relations between them. F_{process} can be efficiently estimated by F_{comp} [25]: $F_{\mathrm{process}} \geq 2F_{\mathrm{comp}} - 1$. In addition, F_{av} can be represented in terms of F_{process} by $F_{\mathrm{av}} = (MF_{\mathrm{process}} + 1)/(M+1)$ [26], where M is the dimension of a quantum gate. Hence one can use F_{comp} to estimate both the fidelities F_{process} and F_{av} by the above connections [21].

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[27] The kernel of the witness operator $\hat{W}_{\mathrm{CZ}}$ is designed as $\hat{\mathcal{W}}_{\mathrm{CZ}} = \sum_{(\alpha,\beta)\in\{0,\pi\},\{\pm(\pi/2)\}} |\alpha_+\rangle_{22}\langle\alpha_+| \otimes |\beta_+\rangle_{33}\langle\beta_+| \otimes \hat{U}_{\mathrm{CZ}}|\mathrm{In}_\beta^\alpha\rangle\langle\mathrm{In}_\beta^\alpha|\hat{U}_{\mathrm{CZ}}^\dagger$, where $\hat{U}_{\mathrm{CZ}} = (\hat{H}\otimes\hat{H})U_{\mathrm{CZ}}$ and U_{CZ} is the two-qubit controlled-Z gate. Here, $|\alpha(\beta)_+\rangle_k = (|0\rangle_k + e^{i\alpha(\beta)}|1\rangle_k)/\sqrt{2}$ determines the input state $|\mathrm{In}_\beta^\alpha\rangle = |-\alpha_+\rangle_2 \otimes |-\beta_+\rangle_3$ for the target gate and shows the one-to-one correspondence between $|\mathrm{In}_\beta^\alpha\rangle$ and the output state $\hat{U}_{\mathrm{CZ}}|\mathrm{In}_\beta^\alpha\rangle$. For the present quantum circuit, F_{comp} is defined by $F_{\mathrm{comp}} \equiv 1/8\sum_{(\alpha,\beta)\in\{0,\pi\},\{\pm(\pi/2)\}} \langle\mathrm{In}_\beta^\alpha|\hat{U}_{\mathrm{CZ}}^\dagger\mathcal{E}_{\mathrm{CZ}}(|\mathrm{In}_\beta^\alpha\rangle\langle\mathrm{In}_\beta^\alpha|)\hat{U}_{\mathrm{CZ}}|\mathrm{In}_\beta^\alpha\rangle$, where $\mathcal{E}_{\mathrm{CZ}}$ denotes the experimental gate operations. If the ideal probability $P(\alpha_+,\beta_+) = 1/4$ is assigned to all the settings of (α,β), we have the relation $F_{\mathrm{comp}} = \langle\hat{\mathcal{W}}_{\mathrm{CZ}}\rangle/2$. See Supplemental Material [21] for detailed discussions.

[28] Four features and implications of the steering witness (1) are summarized as follows. First, the states detected by this witness enable the quantum protocols to be implemented even when untrusted measurement apparatus are unavoidably used (Fig. 1). Second, the state fidelity, $F_S(\rho_{\mathrm{expt}})$, can be estimated from $W_4(\rho_{\mathrm{expt}})$ (4), and $F_S(\rho_{\mathrm{expt}})$ also acts as an indicator showing genuine multipartite steerability. Third, $W_4(\rho_{\mathrm{expt}})$ can be used to estimate the computation fidelity of one-way computation (5), and the steerability can be verified by performing one-way computation as well. Finally, only the minimum two local measurement settings are sufficient to efficiently show the steerability.

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